# HYDRO GENERATION SCHEDULING AND OFFERING STRATEGIES CONSIDERING PRICE UNCERTAINTY AND RISK MANAGEMENT


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**Abstract – Under the current European energy policy towards a sustainable environment, the optimization of the hydropower resources is of crucial importance. In this paper, a mixed-integer nonlinear programming approach is proposed for the short-term hydro scheduling problem, considering head-dependency and discontinuous operating regions. As new contributions to earlier studies, market uncertainty is introduced via price scenarios and risk management is incorporated by limiting the volatility of the expected profit through the conditional value-at-risk. Besides, plant scheduling and pool offering by the hydro power producer are simultaneously considered to solve a realistic hydro system with three cascaded reservoirs. Finally, conclusions are duly drawn.**



***Keywords: Generation, Hydro energy, Pricing, Uncertainty, Risk***


## 1 INTRODUCTION

The renewable technology with the greatest share in electricity generation today in Portugal is of hydro origin. The total installed capacity at end of year 2009 reached 16738 MW, of which 4578 MW (27%) corresponded to hydro plants. Portugal has ambitious goals in terms of renewable energies: 45% of total electricity production by 2010, surpassing 60% by 2020. Hydropower is cost-competitive and can be used as a storage system diminishing the effect of the stochastic wind power. Hence, hydropower is one of the key priorities for the future, aiming to bring Portugal closer to reference countries such as Austria and Sweden. Planned investments will enable Portugal to reach 8600 MW of installed capacity by 2020, significantly increasing the hydropower potential.

In the Portuguese system there are several cascaded hydro systems formed by small reservoirs. This is the situation of the Douro River in the North of Portugal, for instance, which represents about two-thirds of the total hydro generation in the country. In hydro plants with a small storage capacity the operating efficiency is sensitive to the head — head change effect [1]. Hence, it is necessary to consider head-dependency on short-term hydro scheduling (STHS).

In a competitive environment, such as the Norwegian case [2], a power producer has a goal to produce electricity and sell it with maximum profit [3]. The optimal management of the hydropower resources available delivers a self-schedule and represents a major advantage for a hydro power producer to face competition.

STHS models provide decision support for the operational task of bidding in the electricity market [4]. The development of optimal offering strategies is crucial for the hydro power producer to maximize its profit.

Mixed-integer linear programming (MILP) is becoming frequently used for STHS [5]-[8], where integer variables allow modelling of discrete hydro unit-commitment constraints. A nonlinear model has advantages compared with a linear one. The use of nonlinear programming (NLP) in some case studies [9] has lead to an increase in profit of about four percent compared to using linear programming (LP), requiring a negligible extra computation time.

Since the nonlinear model cannot avoid water discharges at forbidden areas, mixed-integer nonlinear programming (MINLP) approaches have been recently proposed in [1,10] to solve the STHS problem. However, the STHS problem was treated as a deterministic one, ignoring uncertainties, which may not be a realistic assumption nowadays.

Day-ahead energy market prices are quite volatile, hard to predict, and subject to data uncertainty caused by non-anticipated market conditions. Price volatility throughout the day can affect notably the profits of the hydro power producer [6]. Moreover, most power producers are, in fact, risk-averse [11]. Hence, to manage risk along with generation scheduling, and to achieve a more uniform profit distribution among scenarios, a risk measure should be taken into account. The optimal self-schedule is then used to derive appropriate offering strategies to the pool.

As new contributions to earlier MINLP models [1,10], market uncertainty is introduced via price scenarios and risk management is incorporated by limiting the volatility of the expected profit through the conditional value-at-risk (CVaR). Besides, plant scheduling and pool offering by the hydro power producer are simultaneously considered to solve a realistic hydro system with three cascaded reservoirs. Finally, conclusions are duly drawn.

## 2 PROBLEM FORMULATION

### *2.1 Risk Management*

CVaR represents an appropriate approach to address risk management for a hydro power producer. Previous MINLP approaches [1,10], however, did not consider risk management.

VaR has the additional difficulty, for stochastic problems, of requiring the use of binary variables for its modelling.

Instead, CVaR computation does not require the use of binary variables and it can be modelled by the simple use of linear constraints. The concept of CVaR is illustrated in Figure 1.

CVaR is the expected profit not exceeding a measure $\zeta$ called Value-at-Risk (VaR):

$$\text{CVaR} = E(B \mid B \le \zeta) \tag{1}$$

VaR is a measure computed as the maximum profit value such that the probability of the profit being lower than or equal to this value is lower than or equal to $1-\delta$, where $\delta$ is the per unit confidence level, i.e.:

$$\text{VaR} = \max\{x \mid \text{p}(B \le x) \le 1-\delta\} \tag{2}$$

The value of $\delta$ is commonly set between 0.90 and 0.99 [12]. In this paper, $\delta$ is considered equal to 0.95. Mathematically, CVaR can be defined as:

$$\max \quad \zeta - \frac{1}{1-\delta}\sum_{n=1}^{N}\rho_n\eta_n \tag{3}$$

subject to:

$$-B_n + \zeta - \eta_n \le 0 \tag{4}$$

$$\eta_n \ge 0 \tag{5}$$

where $\rho_n$ is the occurrence probability of scenario $n$, $\eta_n$ is the auxiliary variable used to compute CVaR, and $B_n$ is the benefit in scenario $n$. Constraints (4) and (5) enforce conditions pertaining to the risk term. In (4), $\eta_n$ is equal to zero if scenario $n$ has a profit greater than $\zeta$. For the remaining scenarios, $\eta_n$ is equal to the difference of $\zeta$ and the corresponding profit.

### 2.2 Objective Function

In the STHS problem under consideration, the objective function takes into account all the price scenarios at once, weighted by their occurrence probability.

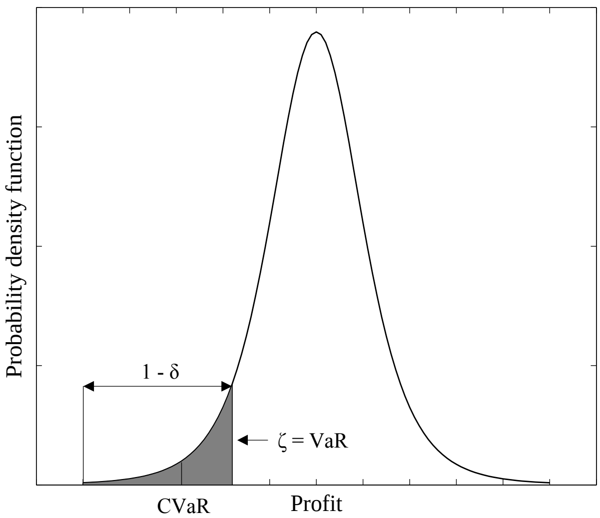


**Figure 1:** VaR and CVaR illustration.

The STHS problem can be formulated as to maximize:

$$J = \sum_{n=1}^{N}\rho_n B_n + \alpha\left(\zeta - \frac{1}{1-\delta}\sum_{n=1}^{N}\rho_n\eta_n\right) \tag{6}$$

The objective function (6) is defined as the expected total profit of the hydro power producer plus a risk measure of the profit. The CVaR approach is included into the formulation providing a trade-off between maximum expected profit and profit volatility. In (6), $\rho_n$ is the probability associated to scenario $n$, $\alpha$ is the weighting positive factor to achieve an appropriate trade-off between profit and risk, which depends on the preferences of the producer, and $\zeta$ is the Value-at-Risk at a confidence level of $\delta$. A risk-averse producer tends to minimize the risk selecting a large value of $\alpha$ to increase the weight of the risk measure in (6). Otherwise, a risk-neutral producer tends to maximize the risk selecting a small value of $\alpha$ to obtain a higher profit. $B_n$ is the benefit for each price scenario, given by:

$$B_n = \lambda_{kn}\sum_{i=1}^{I} p_{ik} \tag{7}$$

where $\lambda_{kn}$ is the energy price for scenario $n$ at the period $k$, and $p_{ik}$ is the power generation of plant $i$ during the period $k$.

### 2.3 Hydro Constraints

The optimal value of the objective function is determined subject to constraints of two kinds: equality constraints and inequality constraints, or simple bounds on the variables. The constraints are indicated as follows:

- Water conservation equation for each reservoir

$$v_{ik} = v_{i,k-1} + a_{ik} - q_{ik} - s_{ik} + q_{i-1,k} + s_{i-1,k} \tag{8}$$

- Power generation equation

$$p_{ik} = P_{ik}(q_{ik}, \eta_{ik}) \tag{9}$$

- Head equation

$$h_{ik} = H_{ik}(l_{ik}, l_{i+1,k}) \tag{10}$$

- Water storage constraints

$$v_i^{\min} \le v_{ik} \le v_i^{\max} \tag{11}$$

- Water discharge constraints

$$u_{ik}\, q_i^{\min} \le q_{ik} \le u_{ik}\, q_i^{\max} \tag{12}$$

- Water spillage constraints

$$s_{ik} \ge 0 \tag{13}$$

Equation (8) corresponds to the water conservation equation for each reservoir, assuming that the time required for water to travel from a reservoir to a reservoir directly downstream is less than the one hour period, independently of water discharge, due to the small distance between consecutive reservoirs.

In (8) $v_{ik}$ is the water storage of reservoir *i* at end of period *k*, $a_{ik}$ is the inflow to reservoir *i* during the period *k*, $q_{ik}$ is the water discharge of plant *i* during the period *k*, and $s_{ik}$ is the water spillage by reservoir *i* during the period *k*.

In (9) power generation, $p_{ik}$, is considered a function of water discharge and efficiency, $\eta_{ik}$, depending on the head, $h_{ik}$. Hence, the electrical output of a hydro plant depends on the water discharge, the head, and the efficiency. The operating points are restricted by minimal and maximal water discharges [13].

In (10) the head is considered a function of the water levels in the upstream reservoir, $l_{ik}$, and of the downstream reservoir, $l_{i+1,k}$, depending on the water storages in the respectively reservoirs.

In (11) water storage has lower and upper bounds. Here for each reservoir *i*, $v_i^{\max}$ is the maximum storage, and $v_i^{\min}$ is the minimum storage.

In (12) water discharge has lower and upper bounds. Here for each reservoir *i*, $q_i^{\max}$ is the maximum discharge, and $q_i^{\min}$ is the minimum discharge. The binary variable, $u_{ik}$, is equal to 1 if plant *i* is on-line in hour *k*, otherwise is equal to 0. In (13) a null lower bound is considered for water spillage. Normally, water spillage by the reservoirs occurs when without it the water storage exceeds its upper bound, so spilling is necessary to avoid damage. The initial water storages, $v_{i0}$, and the inflows to reservoirs are assumed known.

## 3 PROPOSED APPROACH

The MINLP can be stated as to maximize:

$$\boldsymbol{J}(\boldsymbol{x}) \tag{14}$$

subject to:

$$\boldsymbol{x}^{\min} \le \boldsymbol{x} \le \boldsymbol{x}^{\max} \tag{15}$$

$$\boldsymbol{b}^{\min} \le \boldsymbol{A}\,\boldsymbol{x} \le \boldsymbol{b}^{\max} \tag{16}$$

$$\boldsymbol{x}_j \text{ integer} \tag{17}$$

where $\boldsymbol{J}(\cdot)$ is a nonlinear function of the vector $\boldsymbol{x}$ of decision variables, $\boldsymbol{x}^{\min}$ and $\boldsymbol{x}^{\max}$ are the lower and upper bound vectors on variables, $\boldsymbol{A}$ is the constraint matrix, $\boldsymbol{b}^{\min}$ and $\boldsymbol{b}^{\max}$ are the lower and upper bound vectors on constraints. Equality constraints are defined by setting the lower bound equal to the upper bound, i.e. $\boldsymbol{b}^{\min}=\boldsymbol{b}^{\max}$. The variables $\boldsymbol{x}_j$ are restricted to be integers. Note that (6) is rewritten into (14). The water conservation equation (8) is rewritten into (16), as well as the lower and upper bounds for water discharge given in (12). Eq. (15) corresponds to the inequality constraints in (11) and (13).

Our nonlinear objective function is achieved by means of two linearizations: the first of them, efficiency as a function of head, is acceptable; the second one, water level as a function of water storage, implies reservoirs with vertical walls, which however is a good approximation for reservoirs with a small storage capacity, as our data have shown for our case study.

Power generation is considered a nonlinear function of water discharge and water storage, given by:

$$p_{ik} = q_{ik}(\alpha_i \beta_i v_{ik} + \alpha_i l_{i0} - \alpha_i \beta_{i+1} v_{i+1,k} - \alpha_i l_{i+1,0} + \eta_{i0}) \tag{18}$$

A major advantage of our MINLP approach is to consider the head change effect in a single function (18) of water discharge and water storage, which can be used in a straightforward way, instead of deriving several curves for different heads. A MILP approach can be used to find a starting point for the MINLP. Still, initial values do not affect the optimization results.

The model presented in this paper is especially indicated for systems in which the daily policy of water releases has significant influence on the hourly heads [6], i.e., when it is really important to consider the head variation to get optimal or near-optimal realistic schedules, as occurs for instance in Portugal and Spain.

In our model, market uncertainty is introduced in the MINLP model via price scenarios and risk management is incorporated through CVaR. Therefore, the trade-off of maximum profit versus minimum risk is now properly addressed.

The hydro generation scheduling is used to develop appropriate offering strategies to the pool.

## 4 CASE STUDY

The MINLP approach, which considers not only head-dependency and discontinuous operating regions, but also price uncertainty and risk management, has been applied on a case study based on one of the Portuguese cascaded hydro systems.

This approach has been developed and implemented in MATLAB and solved using the optimization solver package Xpress-MP. The numerical simulation has been performed on a 600-MHz-based processor with 256 MB of RAM.

In [1,10], energy prices were considered as deterministic input data. Instead, several prices scenarios are considered in this paper using the neural network approach proposed in [14]. The hydro power producer is considered a price-taker and therefore the price variable is an exogenous parameter for the proposed approach.

The price scenarios over the 24-hours time horizon are shown in Figure 2 (where $ is a symbolic economic quantity). The number of price scenarios generated in the optimization problem is $N = 20$. This number has been selected arbitrarily, and the probability of each generated scenario will be $1/N$.

Final water storage in reservoirs is constrained so the water storage in the reservoirs at the last period is fixed. The final water storage in reservoirs is considered equal to the value at the beginning of the time horizon.

The storage targets for the short-term time horizon established by medium-term planning studies may be represented either by a penalty on water storage or by a previously determined 'future cost function' [15,16].

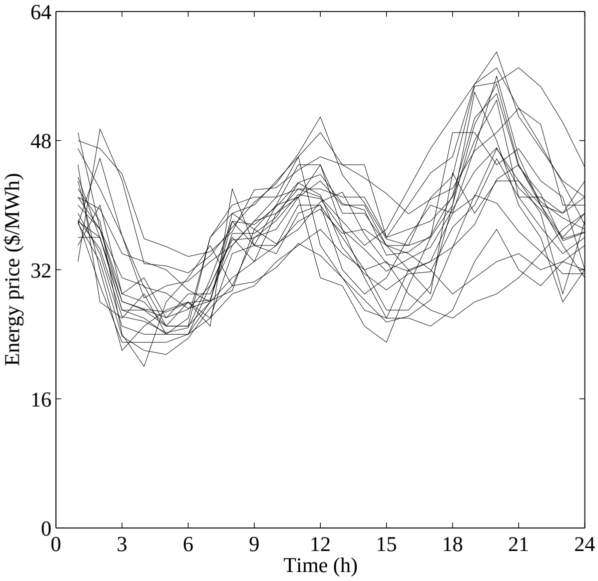


**Figure 2:** Energy price profile considered.

The realistic hydro system with three cascaded reservoirs is shown in Figure 3.

Only the first reservoir has inflow. This inflow is due to an upstream watershed belonging to a different company. The inflow on the first reservoir is shown in Figure 4. Pumping is not available.

The expected profit versus profit standard deviation is presented in Figure 5, considering seven values for $\alpha$. This figure provides the maximum achievable expected profit for each risk level or, alternatively, the minimum achievable risk level for each expected profit.

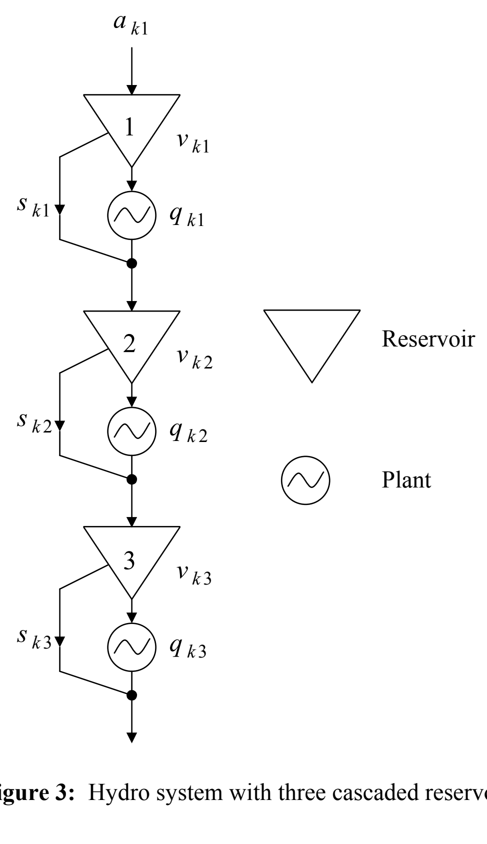


**Figure 3:** Hydro system with three cascaded reservoirs.

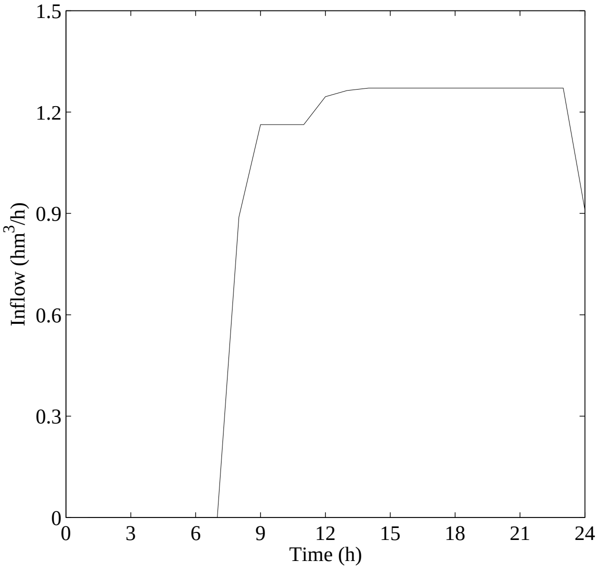


**Figure 4:** Natural inflow on the first reservoir.

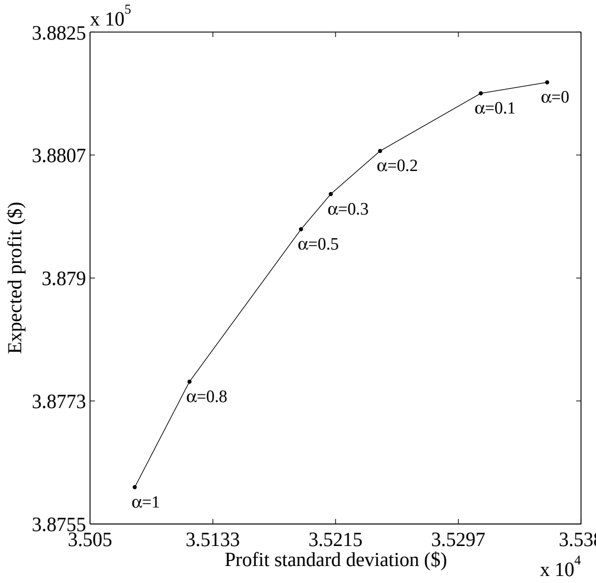


**Figure 5:** Expected profit versus profit standard deviation.

An analysis of Figure 5, known as efficient frontier or Markowitz frontier, reveals that for a risk-neutral producer ($\alpha = 0$), the expected profit is $388178 with a standard deviation of $35357. On the other hand, a risk-averse producer ($\alpha = 1$) expects to achieve a profit of $387602 with a lower standard deviation of $35080. The expected profit results for a risk-averse producer are obtained by considering $\alpha = 1$ in (6), while the expected profit results for a risk-neutral producer are obtained by considering $\alpha = 0$ in (6).

Table 1 establishes a numerical comparison of the increase in profit for several risk levels.

The maximum profit represents an increase of 0.15% corresponding to risk level $\alpha = 0$. Hence, different hydro power producers may choose different behaviours towards risk. Based on the results obtained, risk levels implying $\alpha = 0.2$ or lower are not recommended.

The optimal reservoir storage trajectories are shown in Figure 6. The optimal plant discharge trajectories are shown in Figure 7.

The solid line denotes the results obtained using a risk level $\alpha = 0$, while the dashed line denotes the results obtained using a risk level $\alpha = 1$.

Risk makes possible a different behaviour, especially for the first reservoir, implying that for a risk-neutral producer the influence of the head change effect is more relevant.

The results in Figure 7 are consistent with those in Figure 6. The risk-neutral producer aims at discharging mostly during peak-hours, thus obtaining maximum profit. Instead, by assuming higher values for the risk penalty factors, the number of online hours tends to decrease.

Figures 8 and 9 presents the histograms of the profits for $\alpha = 0$ and $\alpha = 1$, respectively.

| Risk Level | Profit Std. Deviation (€) | Expected Profit (€) | % Increase | CPU Time (s) |
|---|---|---|---|---|
| 1.0 | 35080 | 387602 | - | 2.31 |
| 0.8 | 35116 | 387752 | 0.04 | 2.11 |
| 0.5 | 35191 | 387969 | 0.09 | 2.05 |
| 0.3 | 35211 | 388019 | 0.11 | 1.81 |
| 0.2 | 35244 | 388080 | 0.12 | 1.62 |
| 0.1 | 35312 | 388162 | 0.14 | 1.55 |
| 0.0 | 35357 | 388178 | 0.15 | 1.48 |

**Table 1:** Comparison of the increase in profit for several risk levels.

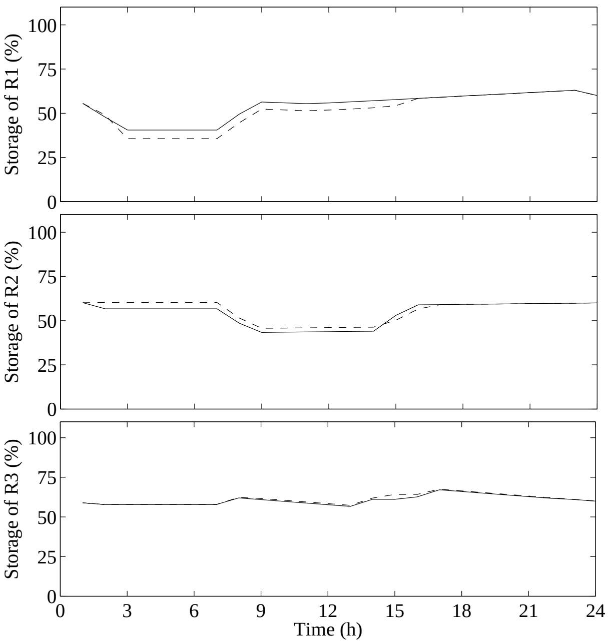


**Figure 6:** Optimal reservoir storage trajectories. The solid line denotes the results obtained considering $\alpha = 0$, while the dashed line denotes the results obtained considering $\alpha = 1$.

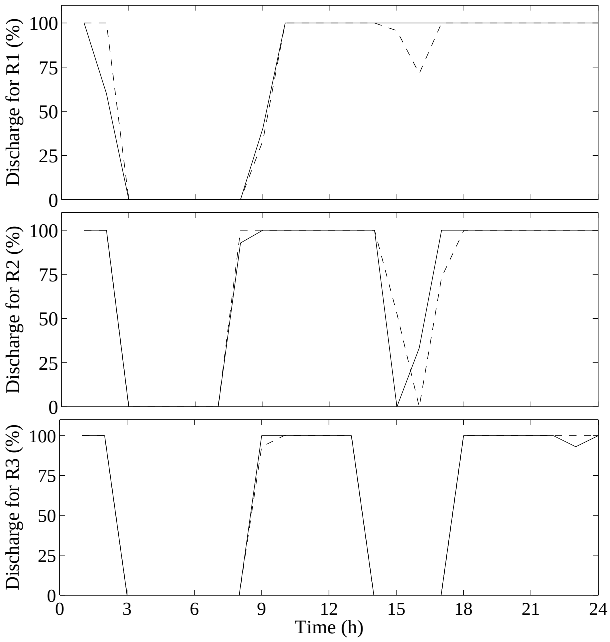


**Figure 7:** Optimal plant discharge trajectories. The solid line denotes the results obtained considering $\alpha = 0$, while the dashed line denotes the results obtained considering $\alpha = 1$.

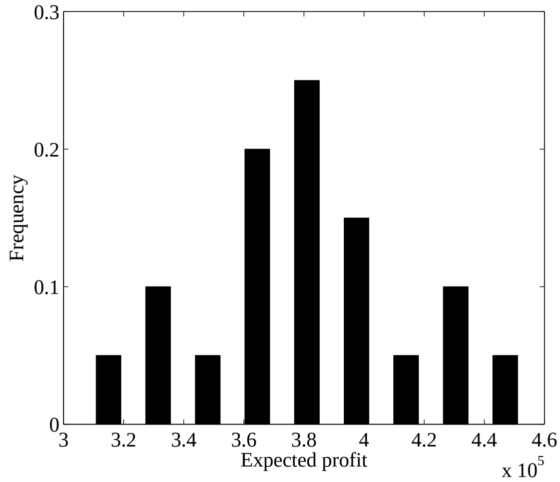


**Figure 8:** Histogram of the profits corresponding to $\alpha = 0$.

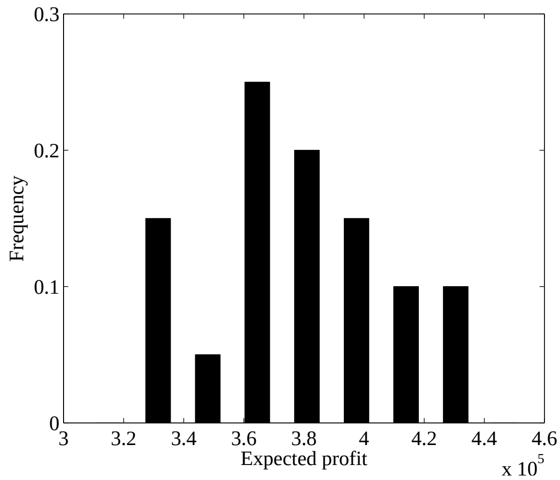


**Figure 9:** Histogram of the profits corresponding to $\alpha = 1$.

Analyzing Figures 8 and 9, it can be verified that the risk level corresponding to $\alpha = 0$ implies a higher expected profit than for $\alpha = 1$. However, $\alpha = 0$ is riskier than $\alpha = 1$, because financial loss can occur under some scenarios. Thus, a risk-averse investor would prefer $\alpha = 1$ because it gives almost the same expected profit level and exhibits lower financial risk. Hence, our model allows the decision maker to obtain solutions according to the desired risk exposure level.

Figure 10 presents the hourly bids (quantity–price pairs) for the hydro system considered in this case study.

The hourly supply functions have to be monotonically increasing functions. The method proposed in this paper for building these supply functions relies on solving independent problems with different final level conditions at each reservoir. In this case study, 11 different final level conditions have been considered, with values ranging from 7 to 77 \$/MWh. Hence, the curves are represented by piecewise linear approximations formed by 11 segments.

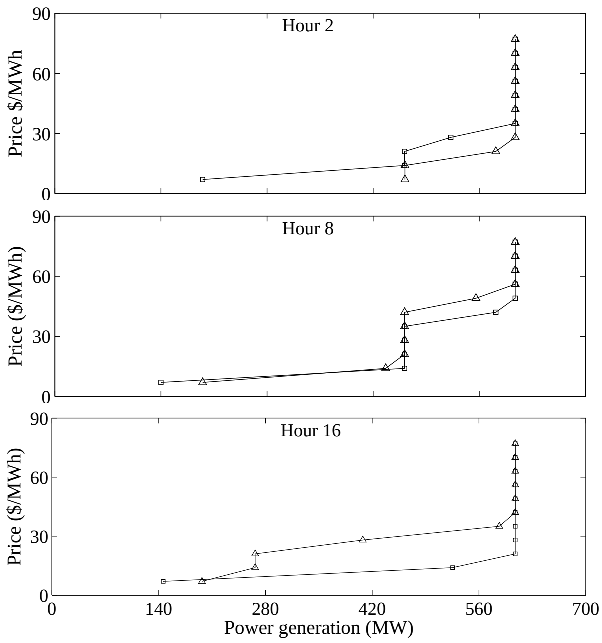


**Figure 10:** Hourly supply functions generated, for the risk levels corresponding to $\alpha = 0$ (□) and $\alpha = 1$ (Δ).

## 5 CONCLUSION

A mixed-integer nonlinear programming approach is proposed for the short-term hydro scheduling problem, considering not only head-dependency and discontinuous operating regions, but also price uncertainty and risk management. The presented results on a realistic case study validate the proficiency of the proposed approach, enabling the selection of the best solution according to the desired risk exposure level, while the average computation time is acceptable. The optimal self-schedule is also used to derive appropriate offering strategies to the pool.


## ACKNOWLEDGMENT

This work is funded by FEDER funds (European Union) through the Operational Programme for Competitiveness Factors – COMPETE, and by Portuguese funds through the Fundação para a Ciência e a Tecnologia – FCT, under Project No. FCOMP-01-0124-FEDER-014887 (Ref. FCT PTDC/EEA-EEL/110102/2009). Also, H.M.I. Pousinho thanks FCT for a Ph.D. grant (SFRH/BD/62965/2009).


## REFERENCES


[1] J. P. S. Catalão, H. M. I. Pousinho and V.M.F. Mendes, "Mixed-Integer Nonlinear Approach for the Optimal Scheduling of a Head-Dependent Hydro Chain", Electric Power Systems Research, Vol. 80, No. 8, pp 935-942, August 2010

[2] O. B. Fosso and M. M. Belsnes, "Short-Term Hydro Scheduling in a Liberalized Power System", POWERCON 2004 Proceedings, November 2004

[3] J. P. S. Catalão, S. J. P. S. Mariano, V. M. F. Mendes and L.A.F.M. Ferreira, "Influence of Price Forecasting on Short-Term Thermal Scheduling with Environmental Concerns", 16th PSCC Proceedings, July 2008

[4] M. M. Belsnes and O. B. Fosso, "Competitive Optimal Hydropower Scheduling from Strategy to Operation", 8th IASTED EuroPES Proceedings, June 2008

[5] A. J. Conejo, J. M. Arroyo, J. Contreras and F. A. Villamor, "Self-Scheduling of a Hydro Producer in a Pool-Based Electricity Market", IEEE Transactions on Power Systems, Vol. 17, No. 4, pp 1265-1272, November 2002

[6] J. García-González, E. Parrilla and A. Mateo, "Risk-Averse Profit-Based Optimal Scheduling of a Hydro-Chain in the Day-Ahead Electricity Market", European Journal of Operational Research, Vol. 181, No. 3, pp 1354-1369, September 2007

[7] A. Borghetti, C. D'Ambrosio, A. Lodi and S. Martello, "An MILP Approach for Short-Term Hydro Scheduling and Unit Commitment with Head-Dependent Reservoir", IEEE Transactions on Power Systems, Vol. 23, No. 3, pp 1115-1124, August 2008

[8] S. Bisanovic, M. Hajro and M. Dlakic, "Hydrothermal Self-Scheduling Problem in a Day-Ahead Electricity Market", Electric Power Systems Research, Vol. 78, No. 9, pp 1579-1596, September 2008

[9] J. P. S. Catalão, S. J. P. S. Mariano, V. M. F. Mendes and L. A. F. M. Ferreira, "Scheduling of Head-Sensitive Cascaded Hydro Systems: A Nonlinear Approach", IEEE Transactions on Power Systems, Vol. 24, No. 1, pp 337-346, February 2009

[10]F. J. Díaz, J. Contreras, J. I. Muñoz and D. Pozo, "Optimal Scheduling of a Price-Taker Cascaded Reservoir System in a Pool-Based Electricity Market", IEEE Transactions on Power Systems, Vol. 26, No. 2, pp 604-615, May 2011

[11]S.-E. Fleten and T. K. Kristoffersen, "Short-Term Hydropower Production Planning by Stochastic Programming", Computers & Operations Research, Vol. 35, No. 8, pp 2656-2671, August 2008

[12]A. J. Conejo, R. García-Bertrand, M. Carrión, A. Caballero and A. Andrés, "Optimal Involvement in Futures Markets of a Power Producer", IEEE Transactions on Power Systems, Vol. 23, No. 2, pp 701-711, May 2008

[13]A. Borghetti, A. Frangioni, F. Lacalandra and C. A. Nucci, "Lagrangian Heuristics Based on Disaggregated Bundle Methods for Hydrothermal Unit Commitment", IEEE Transactions on Power Systems, Vol. 18, No. 1, pp 313-323, February 2003

[14]J. P. S. Catalão, S. J. P. S. Mariano, V. M. F. Mendes and L. A. F. M., Ferreira, "Short-Term Electricity Prices Forecasting in a Competitive Market: A Neural Network Approach", Electric Power Systems Research, Vol. 77, No. 10, pp 1297-1304, August 2007

[15]E. Gil, J. Bustos and H. Rudnick, "Short-Term Hydrothermal Generation Scheduling Model using a Genetic Algorithm", IEEE Transactions on Power Systems, Vol. 18, No. 4, pp 1256-1264, November 2003.

[16]W. Uturbey and A. Simões Costa, "Dynamic Optimal Power Flow Approach to Account for Consumer Response in Short Term Hydrothermal Coordination Studies", IET Generation, Transmission and Distribution, Vol. 1, No. 3, pp 414-421, May 2007