

A Novel Optimization Algorithm Solving Network Reconfiguration

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Abstract—This paper presents a new way to formulate and solve the distribution system optimal reconfiguration problem. The system equations representing the network topology, power flow equation and objective function equation are transformed into an artificial dynamic model formulated by using only differential equations, on the basis of an error function defined into a Lyapunov space domain. Possible inequality constraints are handled by using additional slack variables to transform them into equality constraints. The solution of the optimal reconfiguration problem is obtained at the convergence of the artificial dynamics and is based on dynamic optimal power flow formulation. Starting from a time domain continuous problem, a mixed continuous integer solution in Lyapunov space domain is obtained. The results obtained on a test system are shown and discussed.

Keywords— *Distribution system, reconfiguration, optimization, artificial dynamic system, Lyapunov space, radial configurations.*

I. INTRODUCTION

Distribution network reconfiguration is a classical problem in the electrical engineering field, associated with optimization problems such as optimal reconfiguration in normal operating conditions and in emergency conditions. This paper deals with optimal network reconfiguration in normal conditions, carried out when the distribution system has a weakly-meshed structure and the system operation occurs with radial configurations by opening the redundant branches. For a distribution system with B physical branches (including the redundant ones), N nodes and S supplied nodes, the number of open branches is equal to $A = B - N + S$.

The optimal network reconfiguration problem can be described by using different points of view:

1) The *number of objectives*: typical objective functions adopted in the literature are loss minimization, load balancing, voltage deviation, reliability indicators as the energy not supplied (ENS), and so on. Multi-objective formulations have been formulated as well.

2) The *treatment of the constraints*: the optimization procedure is subject to some operational constraints on nodal voltages, branch currents, the radiality of the network, and others. The constraints may be included or not in the objective

function formulation. If the constraints become part of the objective function, penalized objective function are formulated. Otherwise, the constraints are checked to discard the solutions if there are one or more violations.

3) The *nature of the optimization algorithm*, which can be either deterministic (with well-defined optimization rules) or probability-based (with random components).

In practical optimal reconfiguration problems, the problem is formulated in a discrete environment, using the network configurations as a discrete and enumerable set of solutions. However, the number of possible network configurations may be so high that even the generation of all the configurations could be impracticable. Hence, it is generally not possible to be sure that the globally optimal solution has been found. This fact opens the possibility of testing different solvers to try and determine satisfactory pseudo-optimal solutions. Various methods and meta-heuristics have been tested in the literature, some of which have a formal proof of asymptotic convergence to the global optimum, that is, with convergence guaranteed in an infinite number of iterations. However, in engineering terms there is no certainty to find out the best solution in a finite time or number of calculations. Thus, the development of further tools is an ongoing challenging field of research.

In this paper, a novel approach is proposed to address the discrete optimization problem associated with optimal network reconfiguration. A proof of concept of this approach is provided by considering a single objective problem (reconfiguration at minimum losses). The proposed procedure applies a specific solver based on the work of Torelli et al. [1], who showed how it is possible to convert an optimization problem into an artificial and non-conventional dynamic system. This new dynamic optimization process is moving into a Lyapunov space domain and is based on the convergence of the artificial dynamic system, provided that suitable conditions are satisfied on the formulation of the objective function and on a number of additional variables introduced. The main idea of this paper is to design the artificial dynamic system that describes the optimization process, by introducing the information related to the network reconfiguration with minimum distribution network losses.

The paper is organized as follows. Section II recalls the framework of the optimal reconfiguration methods existing in the literature. In Section III the novel method is explained. An application on a test network is reported in Section IV. The concluding remarks are reported in Section V.

II. OVERVIEW OF THE OPTIMIZATION METHODS FOR DISTRIBUTION NETWORK RECONFIGURATION

The total number of radial configurations that can be extracted from a weakly-meshed network can be computed by using the Kirchhoff's theorem [2]. Even though the total number of radial configurations is known, discovering the entire set of the radial configurations is a hard and time consuming task.

The reconfiguration of the network was investigated by using different approaches [3]-[4]. Early examples of network reconfiguration used deterministic methods, starting from the approach introduced in [5]. Some examples of network losses minimization based on *branch-exchange* methods are reported in [6]-[10]. The *branch-and-bound* procedure based on graph theory concepts [11] has been applied in [12]-[13] to minimize network losses. By assuming some simplifications in the problem formulation, linear programming (LP) tools have been used in network reconfiguration in [14]-[16]. MILP (Mixed-Integer Linear Programming) examples are reported in [17]-[18]. The Tabu search approach has been used for loss minimization in [19]-[20].

To improve the exploration of the space solutions, probability-based method were introduced. A summary of the probability-based methods is presented in [4]. For example, the simulated annealing method and examples of its use for optimal reconfiguration are shown in [21]-[23], particle swarm optimization has been addressed in [24] and the use of ant colony optimization for distribution network reconfiguration is summarized in [25]. A hybrid process involving both simulated annealing and Tabu search is reported in [26]. Some examples of Genetic Algorithms (GA) applied to the reconfiguration problem are shown in [27]-[28]. Examples of penalized objective function with improved Tabu search (with the use of mutation) and simulated annealing are reported in [29]. More recently, new methods of search have been introduced in [30]-[31] considering both network reconfiguration and distributed generation. An approach considering the possibility that the distribution system can be delivered from more than one purchase points with different prices in the reconfiguration of the network for short and medium term is shown in [32]-[33].

III. PROPOSED APPROACH

A. The Torelli's control box (TCB)

The general concept is that a mathematical programming problem can be formulated by using a set of differential algebraic equations, in such a way that the solution of the problem is reached at the equilibrium point [34]. If the solution satisfies suitable properties and belongs to the region of attraction of the initial conditions, it can be guaranteed that the solution point is reached, and as such it is possible to

determine the solution point through the convergence of an artificial dynamic model. The properties to be satisfied refer to the application of the Lyapunov theory. If the artificial dynamic model is stable (or at least asymptotically stable), the solutions obtained in the equilibrium point after the convergence of the dynamics correspond to the solutions of the initial mathematical programming problem. The issue of stability of the artificial dynamic model has been addressed in some papers [1], [35], showing that the artificial dynamic model constructed is asymptotically stable and in particular is quite insensitive to a number of factors that lead to the instability of traditional algorithms applied to solve dynamic problems. This finding has been confirmed by a number of applications to different power system problems, making this artificial dynamic model very interesting as an alternative to other solvers dedicated to mathematical programming problems, not only in the power systems area. In particular, the use of this approach may enable the user avoiding the drawbacks of some numerical algorithms that could fail to converge because of high non-linearities of the first-order conditions [34]. Specific aspects such as the region of attraction of the solutions with respect to the initial conditions are still under investigation.

The characteristics of the solver using the artificial dynamic model make it possible to consider the core of this solver as a general purpose shell that can be interfaced with other modules representing the specific mathematical formulation of the problem to be solved. For this reason, this general purpose shell is called here the *Torelli's Control Box* (TCB) solver.

The problem is based on optimal power flow (OPF) analysis [1] that aims at identifying the optimal asset of the control/decision variables \mathbf{u} that minimize an objective function $f(\mathbf{x}, \mathbf{u})$ subject to a number of nonlinear equality constraints $\mathbf{g}(\mathbf{x}, \mathbf{u})$ and inequality constraints $\mathbf{h}(\mathbf{x}, \mathbf{u})$, being \mathbf{x} the vector of dependent variables. In order to deal with the mathematical details of the TCB formulation, let us consider the following constrained nonlinear programming problem

$$\min_{\mathbf{x}, \mathbf{u}} f(\mathbf{x}, \mathbf{u}) \quad (1)$$

subject to

$$\begin{cases} \mathbf{g}(\mathbf{x}, \mathbf{u}) = 0 \\ \mathbf{h}(\mathbf{x}, \mathbf{u}) = 0 \end{cases} \quad (2)$$

The control/decision variables depend on the specific application domain, including the active power generated by the available generators (for optimal power dispatch), the set points of the primary voltage controllers (for secondary voltage regulation), the optimal location of control/generator resources (for planning studies), and the loading factors (for stability analysis).

The dependent variables include voltage magnitude and phase angle at load (PQ) buses, the voltage phase angle and the reactive power generated at the generation (PV) buses and the active and reactive power generated at the slack bus.

The inequality constraints $\mathbf{h}(\mathbf{x}, \mathbf{u})$ include the minimum and maximum allowable limits for each control/decision variable and for each dependent variable. In addition, the control/decision and the dependent variables should satisfy the power flow equations, which represent part of the equality constraints in (2).

The mathematical background of the novel dynamic OPF formulation is formulated by introducing a scalar function describing the problem objectives and constraints and whose minimum coincides with the OPF solution. This can be obtained by observing that the minimization of the objective function $f(\mathbf{x}, \mathbf{u})$ is equivalent to find the zero's of the following error function:

$$\mathbf{e}_1(\mathbf{x}, \mathbf{u}, \zeta) = f(\mathbf{x}, \mathbf{u}) - \zeta \quad (3)$$

where ζ is an additional unknown variable representing the minimum of the function $f(\mathbf{x}, \mathbf{u})$.

Furthermore, the satisfaction of the equality constraints calls for finding the zero's of the following error functions:

$$\mathbf{e}_2(\mathbf{x}, \mathbf{u}) = \mathbf{g}(\mathbf{x}, \mathbf{u}) \quad (4)$$

As far as the inequality constraints are concerned, they can be converted into equality constraints by the introduction of nonnegative slack variables according to the Interior Point theory [36]-[37]. Consequently, their satisfaction calls for finding the zero's of the following error functions:

$$\begin{cases} \mathbf{e}_3(\mathbf{x}, \mathbf{u}, \mathbf{s}) = \mathbf{h}(\mathbf{x}, \mathbf{u}) + \mathbf{s} - \mathbf{h}_{\max} \\ \mathbf{e}_4(\mathbf{x}, \mathbf{u}, \mathbf{t}) = \mathbf{h}(\mathbf{x}, \mathbf{u}) - \mathbf{t} - \mathbf{h}_{\min} \end{cases} \quad (5)$$

where \mathbf{h}_{\max} and \mathbf{h}_{\min} contain the maximum and minimum values of the variables subject to inequality constraints, respectively, while \mathbf{s} and \mathbf{t} are two additional unknown vectors representing nonnegative slack variables.

Thanks to these statements the OPF problem formalized in (1) and (2) can be addressed by solving the following nonlinear system of equations:

$$\begin{cases} \mathbf{e}_1(\mathbf{x}, \mathbf{u}, \zeta) = 0 \\ \mathbf{e}_2(\mathbf{x}, \mathbf{u}) = \mathbf{0} \\ \mathbf{e}_3(\mathbf{x}, \mathbf{u}, \mathbf{s}) = \mathbf{0} \\ \mathbf{e}_4(\mathbf{x}, \mathbf{u}, \mathbf{t}) = \mathbf{0} \end{cases} \quad (6)$$

that may be expressed in a compact form as:

$$\mathbf{e}(\mathbf{z}) = \mathbf{0} \quad (7)$$

with $\mathbf{e} = \{\mathbf{e}_1(\mathbf{x}, \mathbf{u}, \zeta), \mathbf{e}_2^T(\mathbf{x}, \mathbf{u}), \mathbf{e}_3^T(\mathbf{x}, \mathbf{u}, \mathbf{s}), \mathbf{e}_4^T(\mathbf{x}, \mathbf{u}, \mathbf{t})\}^T$ and $\mathbf{z} = \{\mathbf{x}^T, \mathbf{u}^T, \zeta, \mathbf{s}^T, \mathbf{t}^T\}^T$.

According to this paradigm the OPF problem formalized in (7) can be solved by the unconstrained minimization of the following scalar positive semi-definite function

$$w(\mathbf{z}) = \frac{1}{2} \mathbf{e}^T(\mathbf{z}) \mathbf{e}(\mathbf{z}) \quad (8)$$

In trying to address this issue the traditional solution approaches aim at solving the first-order derivative condition:

$$\frac{dw(\mathbf{z})}{dz} = \left(\frac{d\mathbf{e}(\mathbf{z})}{dz} \right)^T \mathbf{e}(\mathbf{z}) = 0 \quad (9)$$

Solving (9) can be impracticable due to the nonlinear nature of the resulting set of equations, so numerical methods are employed in trying and obtaining a solution that is within an acceptable tolerance.

In this paper we propose an alternative solution paradigm aimed at minimizing (8). The insight is to design a stable artificial dynamic model characterized by equilibrium points corresponding to the minima of $w(\mathbf{z})$ (namely, the OPF solutions). In details, we assume that the components of the vector \mathbf{z} evolve according to proper time dynamics $z_i(t)$, where t is an additional parameter seen as an artificial "time".

Under this assumption, the scalar positive semi-definite objective function $w(\mathbf{z})$ can be considered as a Lyapunov function. Therefore, if we impose that its time derivative is negative-definite or negative-semi-definite along the trajectories $z_i(t)$, then the Lyapunov theorem dictates the existence of asymptotically stable equilibrium points minimizing $w(\mathbf{z})$ and, consequently, solving the OPF problem.

To address this issue let's consider the derivative of (8) with respect to the artificial time:

$$\begin{aligned} \dot{w}(\mathbf{z}(t)) &= \frac{dw(\mathbf{z}(t))}{dt} = [\mathbf{e}(\mathbf{z}(t))]^T \frac{d\mathbf{e}(\mathbf{z}(t))}{dt} = \\ &= [\mathbf{e}(\mathbf{z}(t))]^T \dot{\mathbf{e}}(\mathbf{z}(t)) \end{aligned} \quad (10)$$

where the derivative of the error function vector with respect to the artificial time can be computed by applying the composite derivative rule as follows:

$$\dot{\mathbf{e}}(\mathbf{z}(t)) = \frac{d\mathbf{e}(\mathbf{z}(t))}{dt} = \frac{d\mathbf{e}(\mathbf{z}(t))}{dz} \frac{dz(t)}{dt} = \frac{d\mathbf{e}(\mathbf{z}(t))}{dz} \dot{\mathbf{z}}(t) \quad (11)$$

By combining (10) and (11) we obtain:

$$\dot{w}(\mathbf{z}(t)) = [\mathbf{e}(\mathbf{z}(t))]^T \frac{d\mathbf{e}(\mathbf{z}(t))}{dz} \dot{\mathbf{z}}(t) \quad (12)$$

Therefore, if we assume that $\mathbf{z}(t)$ evolves according to the following dynamic equations:

$$\dot{\mathbf{z}}(t) = -K \left[\frac{dw(\mathbf{z}(t))}{dz} \right]^T = -K \left[\frac{d\mathbf{e}(\mathbf{z}(t))}{dz} \right]^T \mathbf{e}(\mathbf{z}(t)) \quad (13)$$

where K is a positive constant. Then, by combining (12) and (13) we obtain:

$$\dot{w}(\mathbf{z}(t)) = -K [\mathbf{e}(\mathbf{z}(t))]^T \frac{d\mathbf{e}(\mathbf{z}(t))}{dz} \left[\frac{d\mathbf{e}(\mathbf{z}(t))}{dz} \right]^T \mathbf{e}(\mathbf{z}(t)) \quad (14)$$

which is a quadratic form and it is certainly negative-semidefinite. Thanks to this important result we can postulate that if $\mathbf{z}(t)$ evolves according to (13) then the Lyapunov conditions are satisfied (namely, $w(\mathbf{z}(t))$ is positive-semidefinite and its derivative with respect to the artificial

time is negative-semidefinite) and the asymptotic stability of the equilibrium points is guaranteed [34].

The formulation of the TCB solver is available to be used in different contexts. In the next section, it has been used to formulate a new solver for optimal distribution system reconfiguration.

B. TCB-based optimal distribution network reconfiguration

B.1. General notation and variables

The distribution system is characterized by the number of nodes N , the number of branches B , and the number of supply points S . For the purpose of notation and without loss of generality, the a single slack bus is considered here ($S = 1$) numbered as the last node.

The status of each branch is represented by using the vector $\mathbf{b} = [b_1, \dots, b_B]^T \in \mathfrak{R}^{B,1}$, with the particularity that the branch states are generally defined here as real numbers, in order to be included in the artificial dynamics model. The restriction to the branch states to be either 0 or 1 at the end of the optimization process will be included as a specific condition, as indicated below.

Some useful operators are introduced: $\text{rep}(\mathbf{e}, B)$ is the operator used for replicating B times a column vector \mathbf{e} , obtaining a matrix with B identical columns; the operator $\text{exc}(\mathbf{e}, \mathbf{E})$ extracts from a square matrix \mathbf{E} the main diagonal and stores it in the column vector \mathbf{e} ; the operator $\text{diag}(\mathbf{b})$ creates a diagonal matrix with the entries of the vector \mathbf{b} on the diagonal.

The topology of the network is represented by using the full and reduced node-to-branch incidence matrices. By considering all the network nodes (N nodes) we have the full node-to-branch matrix $\mathbf{A} \in \mathfrak{R}^{N,B}$, whose entries contain for each column (branch) the non-zero entries at the network nodes where the branch terminals are located (slack node included). From the entries of the matrix \mathbf{A} , a matrix depending on the branch states indicated with real numbers is constructed as $\mathbf{A}(\mathbf{b}) = \mathbf{A} (\text{rep}(\mathbf{b}, B))^T$. The dependence of the problem variables on the vector \mathbf{b} is remarked in the sequel in the same way. The reduced node-to-branch incidence matrix $\mathbf{A}_R(\mathbf{b}) \in \mathfrak{R}^{N-1,B}$ is obtained by eliminating the last row (slack node) from the matrix $\mathbf{A}(\mathbf{b})$.

The reduced network connection matrix $\mathbf{C}(\mathbf{b}) \in \mathfrak{R}^{N-1,N-1}$ is expressed in the form

$$\mathbf{C}(\mathbf{b}) = \mathbf{A}_R(\mathbf{b}) \mathbf{A}_R(\mathbf{b})^T \quad (15)$$

The branch admittance matrix $\mathbf{Y} \in \mathfrak{R}^{B,B}$ is denoted as

$$\mathbf{Y} = \mathbf{G} + \mathbf{jB} \quad (16)$$

By considering the associated weighted branches variables \mathbf{b} the diagonal branches matrix becomes:

$$\mathbf{Y}_b(\mathbf{b}) = \mathbf{G}_b(\mathbf{b}) + \mathbf{jB}_b(\mathbf{b}) = \mathbf{Y} \text{diag}(\mathbf{b}) \quad (17)$$

Other vectors used in the problem formulation are the following:

- vectors with unity terms associated to the nodes and the branches:

$$\mathbf{w}_B = [1, \dots, 1]^T \in \mathfrak{R}^{B,1} \quad (18)$$

$$\mathbf{w}_{N-1} = [1, \dots, 1]^T \in \mathfrak{R}^{N-1,1} \quad (19)$$

- vector containing the additional slack variables:

$$\mathbf{s} = [s_1, \dots, s_{N-1}]^T \in \mathfrak{R}^{N-1,1} \quad (20)$$

B.2. Components of the error function

The error function \mathbf{e} is composed of a number of terms referring to the equations indicating the network topology, power flow solution, losses and additional constraints:

$$\mathbf{e} = [\boldsymbol{\varepsilon}_1^T, \varepsilon_2, \boldsymbol{\varepsilon}_3^T, \boldsymbol{\varepsilon}_4^T, \varepsilon_5, \boldsymbol{\varepsilon}_6^T]^T \quad (21)$$

corresponding with the following definitions:

- a) *first error function vector (power flow equations)*

Let us consider the power flow equations written as:

$$\mathbf{f}_p = \text{real}(\mathbf{f}_1(\mathbf{v}, \boldsymbol{\theta}, \mathbf{b})) - \mathbf{p} \quad (22)$$

$$\mathbf{f}_q = \text{imag}(\mathbf{f}_1(\mathbf{v}, \boldsymbol{\theta}, \mathbf{b})) - \mathbf{q} \quad (23)$$

where:

$$\mathbf{f}_1(\mathbf{v}, \boldsymbol{\theta}, \mathbf{b}) = \text{diag}(\mathbf{v}) (\mathbf{A} \mathbf{Y}_b(\mathbf{b}) \mathbf{A}^T \mathbf{v})^* \quad (24)$$

$\mathbf{v} \in \mathfrak{R}^{N,1}$ is the vector of the node voltage magnitudes;

$\boldsymbol{\theta} \in \mathfrak{R}^{N,1}$ is the vector of the node voltage angles;

$\mathbf{p} = [P_{G1} - P_{D1}, \dots, P_{GN} - P_{D(N-1)}]^T \in \mathfrak{R}^{N-1,1}$ is the net active power vector, in which P_{Gi} and P_{Di} are the active power generation and load at node i , respectively;

$\mathbf{q} = [Q_{G1} - Q_{D1}, \dots, Q_{GN} - Q_{D(N-1)}]^T \in \mathfrak{R}^{N-1,1}$ is the net reactive power vector, in which Q_{Gi} and Q_{Di} are the reactive power generation and load at node i , respectively.

The components $\boldsymbol{\varepsilon}_1 \in \mathfrak{R}^{2N-2,1}$ of the error function referring to the power flow equations are written as follows:

$$\boldsymbol{\varepsilon}_1 = [\mathbf{f}_p^T, \mathbf{f}_q^T]^T \quad (25)$$

- b) *second error function (number of closed branches)*

In the framework adopted in this paper (with $S = 1$), the branches are represented by considering their state expressed by real numbers in the vector \mathbf{b} , so that the scalar error function $\varepsilon_2 \in \mathfrak{R}^{1,1}$ associated with the number of closed branches is written as

$$\varepsilon_2 = \mathbf{w}_B^T \mathbf{b} - N + 1 \in \mathfrak{R}^{1,1} \quad (26)$$

- c) *third error function vector (branch variables)*

Furthermore, the vector error function $\boldsymbol{\varepsilon}_3 \in \mathfrak{R}^{B,1}$ requiring that the state variables associated with the branches have to be equal to 0 or 1 in the final conditions is written as:

$$\boldsymbol{\varepsilon}_3 = \mathbf{b} - \text{diag}(\mathbf{b})\mathbf{b} \quad (27)$$

- d) *fourth error function vector (network topology)*

In order to guarantee the network connection without leaving any node isolated, an inequality constraint is taken into account, according with which each node has to be connected at least with one other node ($\mathbf{h}_{\min} = \mathbf{w}_{N-1}$). This inequality constraint is transformed into an equality constraints by adding a vector \mathbf{s} containing non-negative slack variables. The corresponding component $\boldsymbol{\varepsilon}_4 \in \mathfrak{R}^{N-1,1}$ of the error function is written as:

$$\boldsymbol{\varepsilon}_4 = \text{exc}(\mathbf{c}(\mathbf{b}), \mathbf{C}(\mathbf{b})) - \mathbf{s} - \mathbf{w}_{N-1} \quad (28)$$

There is no term involving upper limits (\mathbf{h}_{\max} is not used).

- e) *fifth error function vector (distribution network losses)*

The objective function considered in this problem is the total losses (scalar error function): as the losses cannot be reduced to zero, a further unknown ζ is added. The component $\varepsilon_5 \in \mathfrak{R}^{1,1}$ of the error function is written as:

$$\varepsilon_5 = P_L(\mathbf{v}, \boldsymbol{\theta}, \mathbf{b}) - \zeta \quad (29)$$

with:

$$P_L(\mathbf{v}, \boldsymbol{\theta}, \mathbf{b}) = \mathbf{v}^T \text{diag}(\cos(\boldsymbol{\theta})) \mathbf{A} \mathbf{G}_b(\mathbf{b}) \mathbf{A}^T \text{diag}(\cos(\boldsymbol{\theta})) \mathbf{v} + \mathbf{v}^T \text{diag}(\sin(\boldsymbol{\theta})) \mathbf{A} \mathbf{G}_b(\mathbf{b}) \mathbf{A}^T \text{diag}(\sin(\boldsymbol{\theta})) \mathbf{v} \quad (30)$$

f) sixth error function vector (treatment of the nodes with unique connection)

Within the general framework, a mandatory connection is imposed for any node connected to a single node in the network structure (whose connecting branch cannot be open). By introducing the vector $\mathbf{a} \in \mathfrak{R}^{N-1,1}$, whose components are $a_i = 0$ if node i is connected to more than one node, and $a_i = 1$ if node i is connected to a single node, the component $\boldsymbol{\varepsilon}_6 \in \mathfrak{R}^{N-1,1}$ of the error function is expressed as

$$\boldsymbol{\varepsilon}_6 = \text{exc}(\mathbf{c}(\mathbf{b}), \mathbf{C}(\mathbf{b})) - \mathbf{a} \quad (31)$$

B.3. Formulation of the differential equations

The mathematical model has been formulated in order to include only a set of differential equations, in which the variable \mathbf{z} is defined as $\mathbf{z} = \{\mathbf{v}^T, \boldsymbol{\theta}^T, \mathbf{b}^T, \mathbf{s}^T, \zeta\}^T$. By denoting with \mathbf{J} the Jacobian matrix of the power flow equations, the differential equations are formulated as follows:

$$\begin{pmatrix} \dot{\mathbf{v}} \\ \dot{\boldsymbol{\theta}} \end{pmatrix} = -K \mathbf{J}^T \boldsymbol{\varepsilon}_1 - K \boldsymbol{\varepsilon}_4 \begin{pmatrix} \frac{\partial \boldsymbol{\varepsilon}_4}{\partial \mathbf{v}} \\ \frac{\partial \boldsymbol{\varepsilon}_4}{\partial \boldsymbol{\theta}} \end{pmatrix} \quad (32)$$

$$\dot{\mathbf{b}} = -K \left(\left(\frac{\partial \boldsymbol{\varepsilon}_1}{\partial \mathbf{b}} \right)^T \text{rep}(\boldsymbol{\varepsilon}_1, B) + \boldsymbol{\varepsilon}_2 + \left(\frac{\partial \boldsymbol{\varepsilon}_3}{\partial \mathbf{b}} \right)^T \boldsymbol{\varepsilon}_3 + \right.$$

$$\left. \bar{\mathbf{A}}_R^T \text{rep}(\boldsymbol{\varepsilon}_4, B) + \left(\frac{\partial \boldsymbol{\varepsilon}_5}{\partial \mathbf{b}} \right)^T \text{rep}(\boldsymbol{\varepsilon}_5, B) + \left(\frac{\partial \boldsymbol{\varepsilon}_6}{\partial \mathbf{b}} \right)^T \text{rep}(\boldsymbol{\varepsilon}_6, B) \right) \quad (33)$$

$$\dot{\mathbf{s}} = K \boldsymbol{\varepsilon}_4; \mathbf{s}(t) > \mathbf{0} \quad (34)$$

$$\dot{\zeta} = K \boldsymbol{\varepsilon}_5 \quad (35)$$

with $\bar{a}_{ij} \in \bar{\mathbf{A}}_R \mid \bar{a}_{ij} = \text{abs}(a_{ij}) \wedge a_{ij} \in \mathbf{A}_R$.

All the derivatives are explicitly calculated and introduced in the model.

In order to run the procedure, the user has to specify the initial conditions for the power flow voltage magnitudes and angles, the initial values of the branch states \mathbf{b} and the value of the parameter K driving the evolution of the artificial dynamic system. The convergence of the process is obtained when all the components of the vector \mathbf{z} reach the stability condition (i.e., their variation is lower than a specified threshold).

IV. CASE STUDY APPLICATION

A. Network description

The method explained in the previous sections has been applied to the 16-node test network [6], reported in Fig. 1 with re-numbered branches, without shunt capacitors.

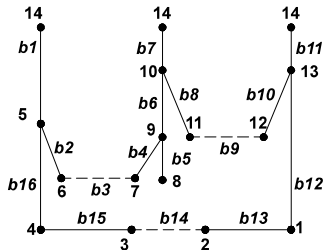


Fig. 1. Structure of the 16-node test network.

In order to construct the node-to-branch incidence matrix, from the topology point of view the three slack nodes are merged into a single point, obtaining $\mathbf{A}_R(\mathbf{b}) \in \mathfrak{R}^{N-1,B}$ with $N = 14$ and $B = 16$. The presence of more slack nodes representing different HV/MV substation can only affect the load flow computation (e.g., the voltages at the substations could be different with each other), but it does not affect the overall optimization procedure. The matrix $\mathbf{A}_R(\mathbf{b})$ is written as follows:

$$\mathbf{A}_R(\mathbf{b}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_{12} & b_{13} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_{13} & b_{14} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_{14} & b_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_{15} & b_{16} & 0 \\ -b_1 & -b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_{16} \\ 0 & b_2 & -b_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_3 & -b_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -b_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_4 & b_5 & -b_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & b_6 & -b_7 & b_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_8 & b_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_9 & b_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_{10} & -b_{11} & b_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (36)$$

The effectiveness of the proposed method has been studied by comparing the results obtained by applying the TCB solver with the results coming from the complete analysis of the 190 different radial configurations of the test network [38]. The best configuration of the network has the open branches b4, b8, and b14, with losses 0.0061 pu. The power flow of the complete solution set has been calculated by using the classical backward-forward sweep (BFS) method.

B. Application of TCB solver by varying the initial conditions on the slack variables

The simulations have been carried out by imposing an initial flat voltage profile (all voltage magnitudes equal to unity and all voltage angles equal to zero), all initial branch states \mathbf{b} equal to unity (i.e., all branches are closed) and the parameter $K = 2 \cdot 10^6$. Moreover, the initial value of the unknown ζ has been determined by considering an arbitrarily high value of the initial losses (equal to 1 pu).

A number of simulations have been carried out by varying the initial conditions on the nodal connections (slack variables \mathbf{s}). The tests have been carried out by filling the initial slack variable vector with values equal to 0, 1 or 2 in a random way. The results have shown that the final configuration reached largely depends on the initial conditions. In a number of cases, the resulting configuration has been one of the six best network configurations. The initial slack variable corresponding with the branch b5 is fixed to unity.

The graphs reported here refer to a case in which the best configuration (global optimum, with open branches b4, b8 and b14) has been found, starting from the slack variable vector containing the values $[1 \ 0 \ 0 \ 1 \ 2 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 2]^T$. The TCB solver, run for an artificial time of $2.5 \cdot 10^{-5}$ artificial time units (atu), finds the globally optimal solution in about 5.5 s (with processor Intel i7, 2.7 GHz, 4 GB of RAM). For this case, the evolution of the losses is reported in Fig. 2.

The evolution of the branch variables, of the voltage magnitudes and of the voltage angles with respect to the artificial time considered are reported in Fig. 3, Fig. 4 and Fig. 5, respectively.

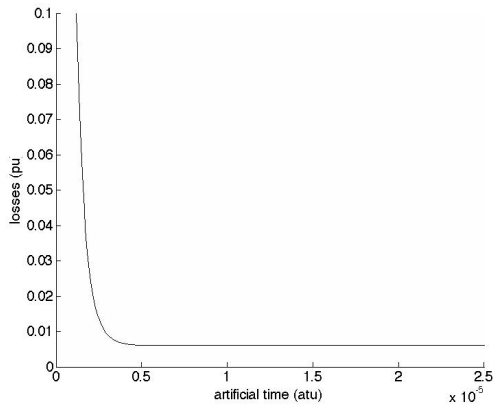


Fig. 2. Evolution of the network losses.

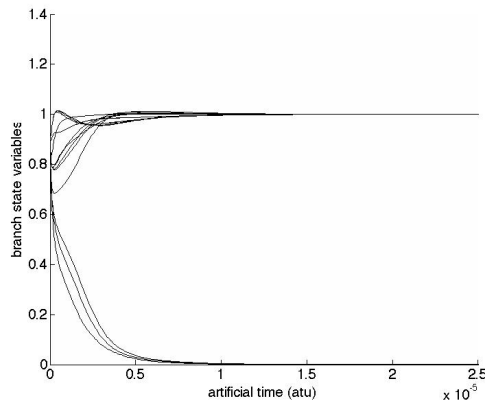


Fig. 3. Evolution of the branch state variables.

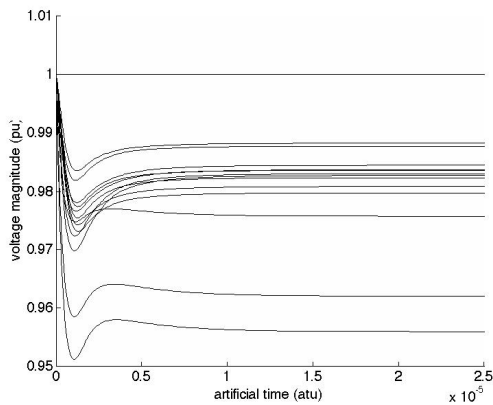


Fig. 4. Evolution of the voltage magnitudes.

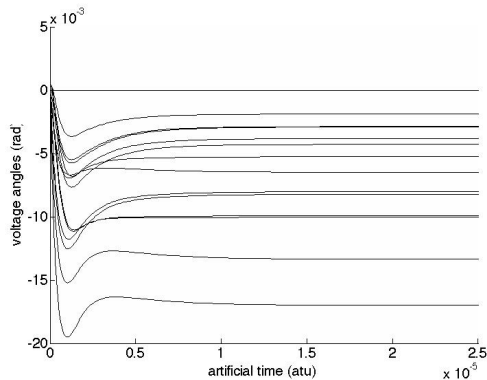


Fig. 5. Evolution of the voltage angles.

TABLE I. NODE VOLTAGES IN THE BEST SOLUTION

node	voltage magnitude (pu)	voltage angle (rad)	node	voltage magnitude (pu)	voltage angle (rad)
1	0.9836	-0.00383	8	0.9560	-0.01700
2	0.9823	-0.00429	9	0.9621	-0.01336
3	0.9797	-0.01003	10	0.9758	-0.00649
4	0.9808	-0.00990	11	0.9837	-0.00291
5	0.9877	-0.00526	12	0.9845	-0.00287
6	0.9830	-0.00802	13	0.9883	-0.00189
7	0.9827	-0.00822	14	1.0000	0

In Fig. 3, the branch state variables of the open branches b4, b8 and b14 converge to zero. Table I shows the node voltages (magnitude and angle) for the solutions of Fig. 4 and Fig. 5.

V. CONCLUSIONS

This paper has provided a proof of concept of the applicability of the TCB to the optimal distribution reconfiguration problem. The characteristics of the problem addressed are different with respect to the ones to which the TCB has been previously applied, because of the presence of discrete variables and of a number of constraints referring to the network topology. The results obtained show that the TCB approach can be effectively used to solve the distribution system reconfiguration problem. The main advantage of the application of the TCB approach is that the solution is found in a fast way, avoiding the need for performing a search on the network topology. For this purpose, it is possible to run the procedure more times to get a number of pseudo-optimal solutions from fast one-shot calculations and maintain the best one found. Future work will address the convergence characteristics of the method, as well as the application to larger systems and multiple objectives.

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