Stabilizing Peer-to-Peer Energy Trading in Prosumer Coalition Through Computational Efficient Pricing

Jing Li[1], Yujian Ye[1,2], and Goran Strbac[1]
[1]Department of Electrical and Electronic Engineering, Imperial College London, London, UK
[2]Fetch.ai, Cambridge, UK
jing.li16@imperial.ac.uk, yujian.ye11@imperial.ac.uk, g.strbac@imperial.ac.uk

***Abstract*—Load balancing issues in distribution networks have emerged alongside the large-scale deployment of distributed renewable generation sources. In light of this challenge, peer-to-peer (P2P) energy trading constitutes a promising approach for delivering secure and economic supply-demand balance when faced with variable load and intermittent renewable generation through matching energy demand and supply locally. However, state-of-the-art mechanisms for governing P2P energy trading either fail to suitably incentivize prosumers to participate in P2P trading or suffer severely from the curse of dimensionality with their computational complexity increase exponentially with the number of prosumers. In this paper, a P2P energy trading mechanism based on cooperative game theory is proposed to establish a grand energy coalition of prosumers and a computationally efficient pricing algorithm is developed to suitably incentivize prosumers for their sustainable participation in the grand coalition. The performance of the proposed algorithm is demonstrated by comparing it to state-of-the-art mechanisms through numerous case studies in a real-world scenario. The superior computational performance of the proposed algorithm is also validated.**

***Index Terms*—Cooperative game theory, distributed energy resources, energy coalition, prosumer, peer-to-peer energy trading.**

I. Introduction

A. Background and Motivation

In recent years, governments across the world have taken significant initiatives towards the decarbonization of energy systems targeted to address environmental and climate change concerns [1]. Alongside which, significant techno-economic challenges emerges primarily associated with the costly balancing of renewable generation and the increase of demand peaks. Aiming at tackling these challenges, a large-scale deployment of distributed energy resources (DER) has been witnessed, including distributed renewable energy sources (RES) and energy storage (ES) units in distribution networks. This enables delivery of the required flexibility to support the cost-effective transition to the low-carbon energy future [2], [3]. This also enhances consumers' ability to harness the energy and turns them to *prosumers* who can actively manage their consumption, generation and storage of energy [3], [4]. To this end, maintaining secure and economic supply-demand balance in the face of variable loads and intermittent RES is of vital importance for the security and reliability of the power system [5], [6].

To encourage self-consumption of RES and therefore alleviate the supply-demand imbalance, the feed-in tariff (FiT) schemes have been put forward worldwide which pay prosumers less for excess generation than they charge for energy consumption. Under such pricing schemes, prosumers with ES units are motivated to store excess generation and discharge later to flatten their peak demand [7]. However, if each prosumer independently optimizes the ES operation (based on its individual demand profile), the joint effect on the net demand profile from multiple ES owners becomes less evident. Alternatively, *peer-to-peer* (P2P) energy trading constitutes a promising approach to encourage the sharing of excess RES energy within a local energy community [8], [9]. P2P energy trading enables coordinated use of complementary DERs, for example photovoltaic (PV) and ES systems, and thus a more locally balanced energy supply and demand. This benefits prosumers economically by enhancing their engagements in system operation by creating a local identity and promoting social cooperation. It also reduces the upstream energy exchange and network losses, deferring/avoiding distribution network reinforcements. Furthermore, P2P trading enables aggregated grouping of small-scale electric loads and renewable sources, reducing their inherent variability as well as increasing local utilization of renewable energy. Recent studies have illustrated these benefits of P2P energy trading [2]–[4].

However, encouraging prosumers to trade energy with one another cooperatively and abstracting the role of the incumbent retailer is a challenging task. It, therefore, calls for an adequate energy trading model which can efficiently manage the local trading among prosumers and design monetary rewards that suitably motivate prosumers to cooperatively participate in P2P trading irrespective of their roles (whether they are electricity buyers or sellers) [10].

B. Relevant Work

In recent years, a considerable amount of research efforts has been made in designing such mechanisms. The research

Submitted to the 21st Power Systems Computation Conference (PSCC 2020).
Jing (Youwen) Li, Yujian Ye, and Goran Strbac are with the Department of Electrical and Electronic Engineering, Imperial College London, London, U.K. (email: jing.li16@imperial.ac.uk, yujian.ye11@imperial.ac.uk, g.strbac@imperial.ac.uk).
The work of Jing (Youwen) Li is sponsored by the Chinese Scholarship Council.

focus of the existing literature can be broadly divided into two categories. In the first category [7], [11]–[13], P2P trading is managed by identifying a set of local trading prices. These include the mid-market rate (MMR) [11], [12] and bill sharing (BS) [7], [13] schemes. However, none of these works considers the participation of prosumers owning ES units, rendering them less relevant for analyzing P2P trading considering the flexibility value of ES to mitigate supply-demand imbalances in the face of intermittent RES and variable loads. Furthermore, both pricing schemes may not be able to suitably incentivize prosumers to participate in P2P trading (as demonstrated in Section V), hindering the successful adoption of P2P trading among prosumers.

In the second category [14], [15], the focus is on discovering benefit distribution mechanisms which can offer prosumers incentives to form *local energy coalitions*. Using *cooperative game theory*, allocation mechanisms based on the *Shapley value* (SV) [14] and the *nucleolus* [15] are introduced. However, SV cannot guarantee a *stabilizing* benefit distribution that ensures no prosumers can benefit more by leaving the *grand coalition* to form smaller coalitions (as demonstrated in Section V). The nucleolus exhibits superior performance with respect to the SV in financially incentivizing prosumers. However, the computation of nucleolus suffers severely from *the curse of dimensionality* as its computational complexity increases exponentially with the number of prosumers [15]. As a result, developing an efficient yet computationally affordable trading and pricing mechanism remains a significant challenge.

C. Contributions

This paper aims at addressing the above limitations of previous approaches. Our novel contributions are outlined as follows:

- A P2P energy trading mechanism is proposed which establishes a prosumer energy grand coalition and computes the highest monetary benefits for prosumers through optimizing the operations of their ES units cooperatively.

- A novel, computationally efficient pricing algorithm is proposed to identify a stabilizing distribution of the total benefits which guarantees prosumers' sustainable participation in P2P trading.

- The favorable computational performance and the effectiveness of the proposed algorithm in promoting prosumers to stay in the grand coalition are demonstrated by comparing it against state-of-the-art mechanisms through numerous case studies in a real-world scenario.

- Results demonstrate that the proposed P2P trading and pricing mechanism is able to optimize the operation of all ES in the coalition cooperatively to minimize the total coalitional energy costs. Prosumers benefit significantly from peak demand reduction and increased RES utilization, and are thus incentivized to share their excess RES generation with their peers.

D. Paper Structure

The remainder of this paper is organized as follows. Section II presents the P2P energy sharing mechanism and the formulation of the cooperative energy management problem. Section III reviews the state-of-the-art pricing and benefit distribution mechanisms. Section IV details the proposed pricing algorithm. Section V presents case studies demonstrating the value of the proposed energy trading and pricing mechanism. Finally, Section V discusses the conclusions and future extensions of this work.

II. P2P Energy Sharing Mechanism and Transactive Energy Management

A. Conventional vs. P2P Energy Trading Paradigms

As discussed in Section I, under the conventional energy trading paradigm, individual *prosumers* trade with the retailer independently based on the offered energy import and export prices (specified in the FiT scheme). However, when each prosumer optimizes the operation of its ES unit independently, according to its own demand and RES output profiles, the joint operation of all ES units is unlikely to yield the minimum total energy cost, rendering a less balanced local supply and demand. As a result, prosumers are motivated to share their surplus generation directly with their neighbors to seek higher revenues, as prescribed by the P2P energy sharing paradigm. In this context, prosumers can first share their generation and consumption internally within an energy coalition and settle the remaining energy deficit or surplus with the retailer.

However, the successful establishment of sustainable prosumer participation in P2P energy trading faces two key challenges: 1) how to optimally schedule the ES units of prosumers and 2) how to design appropriate monetary incentives, which exhibit the *prosumer-centric* property, where it is always beneficial for prosumers to cooperate and trade energy among themselves [16]. To address those challenges, a P2P energy trading and pricing paradigm is proposed which:

i) optimizes cooperatively the charging and discharging schedules of prosumers' ES units by minimizing the total energy cost of all prosumers participating in P2P trading. This coordinates the energy sharing activities among prosumers as well as determines the energy excess/deficit be sold to/bought from the retailer if the local supply cannot balance the demand completely;

ii) calculates the highest energy cost saving that is available to be distributed to the participating prosumers, identifies a set of local trading prices and an associated benefit distribution scheme that adequately maintain prosumers' sustainable participation in P2P trading.

B. Modeling Energy Prosumers

Considering a set of N prosumers, each prosumer $n = 1, 2, ..., N$ is assumed to have no more than one ES unit and one PV system. A trading horizon comprising T time steps ($t = 1, 2, ..., T$) with a temporal resolution of τ is assumed. We consider the ES unit belonging to prosumer n has the minimum and maximum energy limits of $\underline{E}_n$ and $\overline{E}_n$, a charging/discharging limits of $\overline{s}_n$, a charging and discharging efficiency of η_n^c and η_n^d, as well as an initial energy level E_n^0. Following the approach adopted in [17], [18], a generic, technology-agnostic model is employed for the representation of the technical characteristics of the ES unit, which includes:

1)The minimum and maximum energy and power limits for every time step t (1)-(3).

$$\underline{E}_n \leq E_{n,t} \leq \overline{E}_n, \forall n, \forall t, \quad (1)$$

$$0 \leq s^c_{n,t} \leq \overline{s}_n, \forall n, \forall t, \quad (2)$$

$$-\overline{s}_n \leq s^d_{n,t} \leq 0, \forall n, \forall t. \quad (3)$$

2) Energy balance constraint (4): it expresses the electrical energy balance in the ES unit at time step t accounting for the charging and discharging losses.

$$E_{n,t} = E_{n,t-1} + \eta^c_n s^c_{n,t}\tau + s^d_{n,t}\tau/\eta^d_n, \forall n, \forall t. \quad (4)$$

3) Energy neutrality constraint (5): in order to avoid the out-of-horizon effects, a daily periodic continuation is assumed for the operation of ES by assuming equal energy contents at the start and the end of the considered trading horizon.

$$E^0_n = E_{n,T}, \forall n. \quad (5)$$

The net consumption (positive)/generation (negative) $l_{n,t}$ of prosumer n can then be expressed as the summation of the inflexible demand $d_{n,t}$, PV generation $g_{n,t}$, ES charging $s^c_{n,t}$ and discharging $s^d_{n,t}$ power at time period t:

$$l_{n,t} = d_{n,t} + g_{n,t} + s^c_{n,t} + s^d_{n,t}, \forall n, \forall t. \quad (6)$$

At each time period t, prosumer n either acts as a consumer ($l_{n,t} > 0$) or a producer ($l_{n,t} \leq 0$). The set of the consumers and producers are defined in (7) and $\mathcal{N}^c \cap \mathcal{N}^g = \emptyset$.

$$\mathcal{N}^c := \{n \in \mathcal{N} : l_{n,t} > 0\},\ \mathcal{N}^g := \{n \in \mathcal{N} : l_{n,t} \leq 0\}. \quad (7)$$

C. Coalitional Energy Cost and Characteristic Function

The *coalitional game* is applied in [7], [11]–[15] as a P2P trading modeling technique. The concept of *energy coalition* is introduced, where a community of prosumers operate their ES units collaboratively to minimize the total energy cost for the community. For the considered N-prosumer game, the *grand coalition*, denoted by $\mathcal{N} := \{1, 2, ..., N\}$, is constructed when all the prosumers take part in P2P energy trading. Any coalitions formed with the absence of any prosumers are called *sub-coalitions*. For a population of N prosumers, there are in total 2^N possible coalitions can be formed.

We denote the energy import and export prices at each time period t as λ^b_t and λ^s_t, respectively. It follows the common practices in many countries to set the export price in the FiT scheme to be lower than the import price, i.e. $\lambda^b_t > \lambda^s_t, \forall t$. For each coalition $\mathcal{S} \subseteq \mathcal{N}$, the total energy cost $TC(\mathcal{S})$, is defined as the sum of prosumers' energy costs when trading with the retailer in the event of energy imbalance within $\mathcal{S}$:

$$TC(\mathcal{S}) = \sum_{t=1}^{T} \Big(\lambda^b_t \Big[\sum_{n \in \mathcal{S}} l_{n,t}\Big]^+ + \lambda^s_t \Big[\sum_{n \in \mathcal{S}} l_{n,t}\Big]^-\Big), \quad (8)$$

where $[\cdot]^{+/-} = \max / \min\{\cdot, 0\}$. The *coalitional energy cost* is defined as the minimum total energy cost achievable through optimizing the operation of all ES units within coalition $\mathcal{S}$. This optimization problem is formulated as (9) and we define $s^{c*}_{n,t}, s^{d*}_{n,t}, E^*_{n,t}$, and $l^*_{n,t}$ as its optimal solutions.

$$C(\mathcal{S}) = \min_{\{\boldsymbol{s}^c, \boldsymbol{s}^d, \boldsymbol{E}, \boldsymbol{l}\}} TC(\mathcal{S}), \qquad \text{s.t. (1)-(6).} \quad (9)$$

Using *cooperative game theory*, we define the *characteristic function* $v(\mathcal{S}) : 2^{\mathcal{N}} \to \mathbb{R}$ and $v(\emptyset) = 0$ to represent the value of coalition $\mathcal{S}$. In the examined problem, this function quantifies the *energy cost saving* which is expressed as the difference between the sum of the minimum energy cost, where each prosumer in $\mathcal{S}$ independently manages the operation of its ES unit and individually trades with the retailer at the offered energy import/export prices, and the coalitional energy cost of $\mathcal{S}$ by collectively optimizing the management of all prosumers' ES units, as defined in (10). The value of the grand coalition $v(\mathcal{N})$ signifies the highest monetary benefit that is available to be distributed to the N participating prosumers.

$$v(\mathcal{S}) = \sum_{n \in \mathcal{S}} C(\{n\}) - C(\mathcal{S}), \forall \mathcal{S} \subseteq \mathcal{N}. \quad (10)$$

III. State-of-the-art Pricing and Benefit Distribution Mechanisms

As discussed in Section I, different pricing and benefit distribution mechanisms have been developed in the literature. Among them, the most widely recognized methods are MMR, BS, SV and the nucleolus. However, these methods either fail to maintain the sustainable participation of prosumers in P2P energy trading or suffer from *the curse of dimensionality* and correspond to the significant computational burden. Furthermore, the implementation of MMR and BS neglect the participation of prosumers owning ES units with time-coupling operating characteristics. In the view of such limitations, this section presents a novel computationally efficient pricing algorithm based on the concepts of cooperative game theory. Section III-A introduces the core of a game, which consists of benefit distribution methods that can incentivize all participants to stay in the grand coalition. Section III-B to Section III-D compare the aforementioned four state-of-the-art pricing and benefit distribution mechanisms.

A. Core of the P2P Energy Sharing Coalitional Game

We define $\boldsymbol{r} \in \mathbb{R}^N$ as the benefit distribution vector associated with the grand coalition, whose element r_n represents the benefit distributed to prosumer $n \in \mathcal{N}$. $\boldsymbol{r}$ is said to be an *imputation* if it satisfies: i) the criteria of *collective rationality*, which indicates that the highest monetary benefit $v(\mathcal{N})$, namely the energy cost saving of the grand coalition, must be completely allocated to the N participating prosumers, i.e. $\sum_{n \in \mathcal{N}} r_n = v(\mathcal{N})$, and ii) the criteria of *individual rationality* which requires that the distribution to any prosumer n indicated by the benefit distribution vector $\boldsymbol{r}$ to be at least the amount prosumer n can attain on his own, i.e. $r_n \geq 0$[1], $\forall n \in \mathcal{N}$.

[1]The characteristic function represents the energy cost saving associated with a coalition as defined in (10), for the coalition with any single prosumer, $v(\{n\}) = C(\{n\}) - C(\{n\}) = 0, \forall n \in \mathcal{N}$, and the individual rationality is equivalent to $r_n \geq 0, \forall n \in \mathcal{N}$.

$\mathcal{I}$ indicates the set of all imputations for the examined N-prosumer cooperative game:

$$\mathcal{I} := \Big\{ \boldsymbol{r} \in \mathbb{R}^N : \sum_{n \in \mathcal{N}} r_n = v(\mathcal{N}), r_n \geq 0, \forall n \in \mathcal{N} \Big\}. \quad (11)$$

Given some imputation $\boldsymbol{r}$, if there exists a sub-coalition $\mathcal{S} \subset \mathcal{N}$ whose total benefit distribution from $\boldsymbol{r}$ is less than that this sub-coalition can achieve by itself, that is, if $\sum_{n \in \mathcal{S}} r_n < v(\mathcal{S})$, then a tendency emerges for prosumers in $\mathcal{S}$ leaving $\mathcal{N}$ and forming a sub-coalition, since the sub-coalition could guarantee its participants a higher total benefit than they would receive from $\boldsymbol{r}$, and the considered imputation $\boldsymbol{r}$ is thus said to be *unstable*. To this end, the *core* is a set of *stable imputations* which incentivize all prosumers to stay in the grand coalition as breaking off from which to form smaller sub-coalitions only results in dissatisfaction of some prosumers.

$$\mathcal{C} := \Big\{ \boldsymbol{r} \in \mathcal{I} : \sum_{n \in \mathcal{S}} r_n \geq v(\mathcal{S}), \forall \mathcal{S} \subset \mathcal{N} \Big\}. \quad (12)$$

We also introduce the concept of *excess* $e(\mathcal{S}, \boldsymbol{r})$ to measure the dissatisfaction level of coalition $\mathcal{S}$ with respect to imputation $\boldsymbol{r}$. It is defined as the difference between a coalition's energy cost saving and the sum of its participating prosumers' benefit distribution, as defined in (13). It can be observed that the core represents a subset of imputation whose excess of every coalition is non-positive.

$$e(\mathcal{S}, \boldsymbol{r}) = v(\mathcal{S}) - \sum_{n \in S} r_n, \ \forall S \subseteq \mathcal{N}. \quad (13)$$

B. Mid-Market Rate (MMR)

As discussed in Section II-A, under the P2P energy trading paradigm, prosumers first share their generation and consumption among themselves within the coalition at a local trading price and then trade with the retailer to cover the remaining electricity deficit or surplus. The local price should generally be set to be lower than the import price and higher than the export price so that all the prosumers are incentivized to participate in P2P sharing regardless of their roles (buyers or sellers). We define the net consumption P_t^{nc} and net generation P_t^{ng} of the prosumer coalition at time period t and the remaining electricity deficit (positive) and surplus (negative) P_t^{re} at time period t as:

$$P_t^{nc} = \sum_{n \in \mathcal{N}^c} l_{n,t}, \ P_t^{ng} = \sum_{n \in \mathcal{N}^g} l_{n,t}, \ P_t^{re} = \sum_{n \in \mathcal{N}} l_{n,t}, \forall t. \quad (14)$$

The MMR method sets the local buy and sell prices as the average of the retail import and export prices λ_t^{mid} with some adjustments on the basis of the difference between the net consumption and generation of the prosumer coalition. More specifically, at time period t:

1) : If the consumption of the prosumer coalition matches its generation (i.e. $P_t^{re} = 0$), then the local buy and sell prices are set equal to λ_t^{mid}, which is defined as:

$$\lambda_t^{L,b} = \lambda_t^{L,s} = \lambda_t^{mid} = (\lambda_t^b + \lambda_t^s)/2. \quad (15)$$

2) : In case of net consumption of the prosumer coalition (i.e. $P_t^{re} > 0$), the deficit electricity is bought from the retailer at its import price λ_t^b. Since $\lambda_t^b > \lambda_t^{mid}$, an extra payment of the amount $\lambda_t^b P_t^{re}$ will be made and the overall payment is proportionally shared among consumers according to their net consumption $l_{n,t}$. In this case, prosumers will be paid at λ_t^{mid} but pay at a higher local buy price, which is calculated as:

$$\lambda_t^{L,b} = \big(\lambda_t^{mid} |P_t^{ng}| + \lambda_t^b P_t^{re}\big)/P_t^{nc}. \quad (16)$$

3) : In case of net generation of the prosumer coalition (i.e. $P_t^{re} \leq 0$), the surplus electricity is sold to the retailer at its export price λ_t^s. Since $\lambda_t^s < \lambda_t^{mid}$, a revenue shortfall of the amount $\lambda_t^s |P_t^{re}|$ will emerge and the overall shortfall is proportionally shared among producers according to their net generation $|l_{n,t}|$. In this case, the prosumers will pay at λ_t^{mid} but be paid at a lower local sell price, which is calculated as:

$$\lambda_t^{L,s} = \big(\lambda_t^{mid} P_t^{nc} + \lambda_t^s |P_t^{re}|\big)/|P_t^{ng}|. \quad (17)$$

Under the MMR pricing scheme, the energy cost C_n^{MMR} of each prosumer can be calculated by summing up its payments (when $l_{n,t}^* > 0$) and revenues (when $l_{n,t}^* < 0$) using the optimal solution $l_{n,t}^*$ of the optimization problem (9) over the considered time horizon as:

$$C_n^{MMR} = \sum_{t=1}^{T} \big(\lambda_t^{L,b} [l_{n,t}^*]^+ + \lambda_t^{L,s} [l_{n,t}^*]^-\big), \forall n \in \mathcal{N}. \quad (18)$$

The benefit distribution of prosumer n under the MMR pricing scheme can then be expressed as the difference between the minimum energy cost when prosumer n individually optimizes the operation of its ES unit and trade with the retailer (i.e. solve problem (9) by substituting $\mathcal{S}$ with $\{n\}$) and C_n^{MMR}:

$$r_n^{MMR} = C(\{n\}) - C_n^{MMR}, \forall n \in \mathcal{N}. \quad (19)$$

C. Bill Sharing (BS)

BS distributes the total coalitional deficit electricity consumption payment $\sum_{t=1}^T \lambda_t^b [P_t^{re}]^+$/surplus electricity generation revenue $\sum_{t=1}^T \lambda_t^s |[P_t^{re}]^-|$ to individual prosumers according to the proportion of their net energy consumption/energy generation in those of the whole community on a pro rata basis. As such, the local buy and sell prices are set as:

$$\lambda_t^{L,b} = \Big(\sum_{t=1}^{T} \lambda_t^b [P_t^{re}]^+\Big) \Big/ \Big(\sum_{t=1}^{T} P_t^{nc}\Big), \forall t, \quad (20)$$

$$\lambda_t^{L,s} = \Big(\sum_{t=1}^{T} \lambda_t^s |[P_t^{re}]^-|\Big) \Big/ \Big(\sum_{t=1}^{T} |P_t^{ng}|\Big), \forall t. \quad (21)$$

Analogously, under the BS pricing mechanism, the energy cost C_n^{BS} and the corresponding benefit distribution of each prosumer n can be expressed as:

$$C_n^{BS} = \sum_{t=1}^{T} \big(\lambda_t^{L,b} [l_{n,t}^*]^+ + \lambda_t^{L,s} [l_{n,t}^*]^-\big), \forall n \in \mathcal{N}, \quad (22)$$

$$r_n^{BS} = C(\{n\}) - C_n^{BS}, \forall n \in \mathcal{N}. \quad (23)$$

Note that, as opposed to the MMR prices, the BS prices may not be able to incentivize prosumers to participate in P2P energy sharing. For example, under the FiT scheme, the export price is usually fixed for all periods of the day (i.e. $\lambda_t^s = \lambda^s, \forall t$), as a result, equation (21) is simplified to (24). Since $|[P_t^{re}]^-| \leq |P_t^{ng}|, \forall t$, the local sell price $\lambda_t^{L,s}$ can be lower than the export price λ^s, which suggests that prosumers acting as electricity sellers may experience a revenue deficit when participating in the P2P sharing.

$$\lambda_t^{L,s} = \lambda^s \Big(\sum_{t=1}^{T} |[P_t^{re}]^-| \Big/ \sum_{t=1}^{T} |P_t^{ng}| \Big), \forall t. \tag{24}$$

Furthermore, although the implementation of the MMR and BS pricing schemes merits simplicity, the resultant reward distribution solutions are not guaranteed to be in the core of our proposed prosumer coalitional game, as will be further demonstrated in Section V, which means some prosumers may find it more beneficial to defect from the grand coalition.

D. Shapley Value (SV)

The SV [19] is a solution of a cooperative game that prescribes a unique distribution $r_n^{SV} = \phi(\mathcal{N}, v)$ of the total monetary benefit of the grand coalition to each player.

According to the definition of SV [19], the benefit that player n receives in a cooperative game is:

$$\phi(\mathcal{N}, v) = \sum_{\mathcal{S} \subseteq \mathcal{N} \setminus \{n\}} \frac{|\mathcal{S}|!(N - |\mathcal{S}| - 1)!}{N!} \Big(v(\mathcal{S} \cup \{n\}) - v(\mathcal{S}) \Big). \tag{25}$$

where $|\mathcal{S}|$ denotes the bumber of prosumers in coalition $\mathcal{S}$ and $v(\mathcal{S} \cup \{n\}) - v(\mathcal{S})$ represents the *marginal contribution* of prosumer n to coalition $\mathcal{S} \cup \{n\}$. The weight associated with the marginal contribution is the probability that prosumer n joins $\mathcal{S} \cup \{n\}$ right after prosumers in $\mathcal{S}$. The SV expresses the *average marginal contribution* of prosumer n, averaged over all the different permutations in which the grand coalition can be constructed from the empty coalition $\emptyset$. In other words, the SV rewards each prosumer by its marginal contribution to each coalition; the more prosumers contribute the more benefit they receive.

After a complete list of coalition values $v(\mathcal{S}), \forall \mathcal{S} \subseteq \mathcal{N}$ is calculated using (10), the benefit distribution under SV can be easily calculated by (25). Owning to the closed-form representation of the benefit distribution in (25), SV merits computational efficiency. However, as demonstrated in [15], for a game with a non-empty core, the SV solution does not always belong to the core, as also demonstrated in Section V.

E. Nucleolus

The *Nucleolus* [20] constitutes a solution concept targeted to obtain a unique imputation in the core. For the considered coalitional game, let $\boldsymbol{\theta}(\boldsymbol{r}) \in \mathbb{R}^{2^N - 2}$ denote the vector whose entries represent the excesses (i.e. the energy coalition's dissatisfaction level) of all coalitions (excluding the empty coalition $\emptyset$ and the grand coalition $\mathcal{N}$), arranged in a non-increasing order, i.e. the value of the i^{th} entry is always larger than or equal to that of the j^{th} entry when $i \leq j$:

$$\theta(\boldsymbol{r})_i \geq \theta(\boldsymbol{r})_j, \forall i \leq j. \tag{26}$$

Two imputations $\boldsymbol{r}$ and $\boldsymbol{k}$ are said to be ordered *lexicographically* if $\theta(\boldsymbol{r})_i = \theta(\boldsymbol{k})_i$ for $1 \leq i < j$ and $\theta(\boldsymbol{r})_j < \theta(\boldsymbol{k})_j$ for a certain j, which is denoted by $\boldsymbol{r} <_l \boldsymbol{k}$. The nucleolus $\boldsymbol{\nu}$, where $r_n^{Nu} = \nu_n, \forall n \in \mathcal{N}$, represents the lexicographically minimal imputation which minimizes the excesses of all possible coalitions:

$$\boldsymbol{\nu} = \{\boldsymbol{\nu} \in \mathcal{I} : \boldsymbol{\nu} <_l \boldsymbol{r}, \forall \boldsymbol{r} \in \mathcal{I} \setminus \boldsymbol{\nu}\}. \tag{27}$$

The existence of the nucleolus is guaranteed, and for a game with a non-empty core, the nucleolus always lies in the core [15]. The computation of the nucleolus share the same prerequisite of calculating the complete list of coalition values $v(\mathcal{S}), \forall \mathcal{S} \subseteq \mathcal{N}$. Beyond that, it necessitates the solution of $\mathcal{O}(2^N)$ linear programming (LP) problems iteratively (and in the extreme case, it requires solving $2^N - 2$ LPs) [15], prohibiting efficient computation for a game with a large number of prosumers.

IV. Proposed Pricing Algorithm

To address the limitations of previous proposed mechanisms, we propose a novel pricing algorithm which shares the similar computational burden of the SV method while guarantees the resultant benefit distribution belongs to the core of the investigated prosumer coalitional game. We define $\lambda_t^{P,b}$ and $\lambda_t^{P,s}$ as the decision variables prescribed by the proposed pricing algorithm representing the local buy and sell prices of the prosumer coalition at time period t. Then, the benefit distribution r_n^P to each prosumer is equal to the difference between its minimum energy cost by individually trading with the retailer, and the minimum cost achieved by trading locally within the grand coalition at the proposed buy and sell prices:

$$r_n^P = C(\{n\}) - \sum_{t=1}^{T} \big(\lambda_t^{P,b} [l_{n,t}^*]^+ + \lambda_t^{P,s} [l_{n,t}^*]^- \big). \tag{28}$$

Pursing a stable benefit distribution solution which guarantees the sustainable participation of prosumers in the grand coalition and at the same time overcomes the computational challenge of the nucleolus, we identify the local trading prices by first calculating a complete list of coalition values $v(\mathcal{S}), \forall \mathcal{S} \subseteq \mathcal{N}$ using (10) (as did in SV and Nucleolus) followed by the solution of a single LP, which is formulated as follows:

$$\omega^* = \min_{\{\boldsymbol{\lambda}^{P,b}, \boldsymbol{\lambda}^{P,s}, \omega, , \boldsymbol{r}^P \in \mathcal{I}\}} \omega \tag{29a}$$

$$\text{s.t.:} \quad \sum_{n \in \mathcal{N}} r_n^P = v(\mathcal{N}) \tag{29b}$$

$$v(\mathcal{S}) - \sum_{n \in \mathcal{S}} r_n^P \leq \omega, \forall S \subset \mathcal{N} \setminus \emptyset, \tag{29c}$$

$$\omega \leq 0 \tag{29d}$$

$$\lambda_t^s \leq \lambda_t^{P,s} \leq \lambda_t^{P,b} \leq \lambda_t^b, \forall t. \tag{29e}$$

The objective function (29a) minimizes the greatest excess ω of all sub-coalitions (excluding the empty coalition $\emptyset$) by searching through all the possible imputations. Constraint (29b) ensures that the benefit distribution vector $\boldsymbol{r}^P$ satisfies the criterion of collective rationality. Constraint (29c) sets ω as the greatest excess of all sub-coalitions and constraint (29d) ensures the non-positivity of ω. Constraint (29e) ensures that the local buy and sell prices are between the retailer's import and export prices, as such prosumers are incentivized to remain in the grand coalition irrespective of their trading roles.

It is clear that the optimal imputation $\boldsymbol{r}^{P*}$ achieving the minimum ω^* is both collectively and individually rational. From (29c) and (29d), it suffices to see $v(\mathcal{S}) - \sum_{n \in \mathcal{S}} r_n^{P*} \leq \omega^* \leq 0, \forall S \subset \mathcal{N} \backslash \emptyset$. This suggests that the obtained distribution vector $\boldsymbol{r}^{P*}$ belongs to the core, therefore, it guarantees that no prosumer has any incentives to exit the grand coalition.

V. Case Studies

A. Test System and Implementation

In this section, we validate the performance of the proposed P2P energy trading mechanism in a real-world scenario using home solar PV and load data published by Ausgrid, Australia [21]. We validate the effectiveness of the proposed pricing algorithm in sustaining stable participation of prosumers in P2P trading by comparing it against state-of-the-art pricing and benefit distribution mechanisms. A daily trading horizon with an hourly resolution is assumed. We consider the ES system of each prosumer n has a maximum energy limit within range $\overline{E}_n \in [7, 14]$ kWh, a minimum energy limit of $\underline{E}_n = 0.1\overline{E}_n$, a charging/discharging limit of $\overline{s}_n = 0.4\overline{E}_n/\tau$, a charging and discharging efficiency of $\eta_n^c = \eta_n^d = 0.95$ and an initial energy level $E_n^0 = 0.3\overline{E}_n$. The retail import price follows the UK Economy 7 residential rate plan: 7 pence/kWh for 12am-7am and 14.71 pence/kWh for 7am-12am [22], and the retail export price is set as the UK FiT [23] fixed at 4.03 pence/kWh.

All the examined algorithms are implemented in FICO™ Xpress [24] on a computer with a 6-core 3.47 GHz Intel(R) Xeon(R) X5690 processor and 192 GB of RAM.

B. Performance Evaluation

The aim of this section lies in evaluating the performance of the proposed pricing algorithm in financially incentivizing prosumers not to defect from the grand coalition, by comparing it against state-of-the-art mechanisms (i.e. MMR, BS, SV and the nucleolus). To facilitate our analysis, we examine a 4-prosumer coalitional game where 2 ES units and 2 PV systems are randomly assigned to prosumers. Fig. 1 illustrates the distribution of the total monetary benefit to each of the four prosumers under the proposed and state-of-the-art mechanisms. It can be observed that all four prosumers receive positive benefits under SV, the nucleolus and the proposed mechanisms whereas prosumer 4 receives a negative benefit under MMR, prosumer 1 and 3 receive negative benefits under BS. In other words, the energy cost of those prosumers will be lower on their own than participating in P2P energy trading.

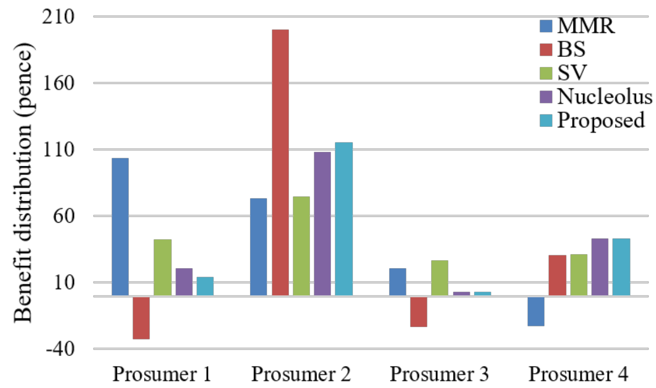

Fig. 1. Benefit distribution of the total monetary benefit for each of the 4 prosumers under the proposed and state-of-the-art mechanisms.

Fig. 2 illustrates the excesses of all sub-coalitions (excluding the empty coalition $\emptyset$) of the examined 4-prosumer coalitional game with respect to the benefit distribution solutions of the proposed and state-of-the-art mechanisms. It can be observed that the excess of every sub-coalition is negative under the benefit distribution of the nucleolus and the proposed pricing algorithm, meaning that all prosumers are satisfied to remain in the grand coalition instead of forming smaller sub-coalitions. In other words, both benefit distribution solutions lie in the core of the coalitional game. On the other hand, the excesses of at least 2 sub-coalitions with respect to the benefit distribution solution of MMR, BS, and SV are greater than zero, suggesting that prosumers breaking off from the grand coalition to form sub-coalitions can achieve higher benefits, resulting in an unstable benefit allocation.

C. Comparison of Pricing Algorithms

Fig. 3 (a)-(c) illustrate the grid import and export prices as well as the local buy and sell prices obtained by MMR, BS, and the proposed pricing algorithm, respectively. It is evident that the local buy and sell prices under MMR and the proposed algorithm are lower than the import price and higher than the export price (Fig. 3 (a) and (c)), suggesting that prosumers all have incentives to participate in P2P energy sharing irrespective of their roles (buyers or sellers), rather than trading independently with the retailer. On the other hand, the local sell prices under BS are lower than the export prices (Fig. 3 (b)). The reason behind this is explained in Section III-C. As a result, prosumers acting as electricity sellers will experience a revenue deficit when participating in P2P energy sharing which is particularly problematic for prosumers who have abundant local generation.

D. Local Demand-Supply Balancing and RES Absorption

The aim of this section lies in comparing the local demand-supply balancing and RES absorption effects associated with the conventional and the proposed P2P energy trading mechanisms discussed in Section II-A. We examine a 16-prosumer coalitional game where 8 ES units and 8 PV systems are randomly assigned to prosumers. Fig. 4 (a) illustrates the net load (positive) and generation (negative) of the 16-prosumer

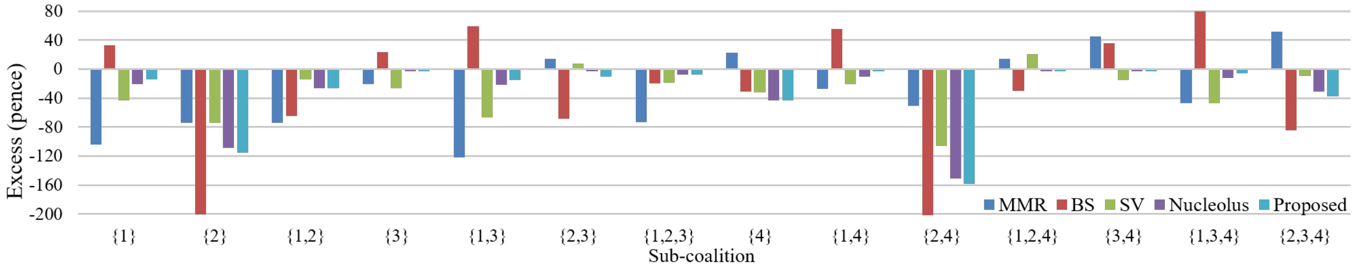

Fig. 2. Excesses of all sub-coalition with respect to the benefit distribution solution under the proposed and state-of-the-art mechanisms for the examined 4-prosumer coalitional game.

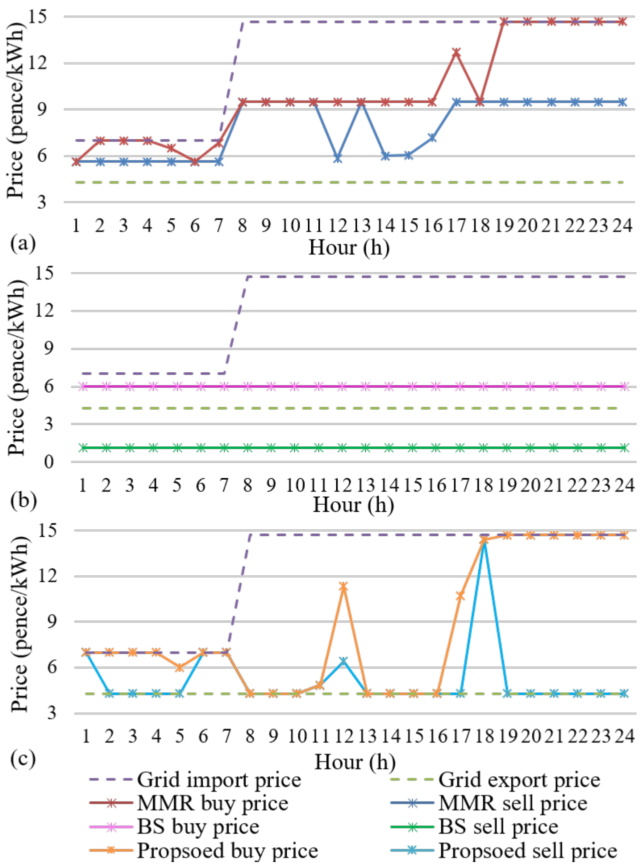

Fig. 3. Comparison of hourly local buy and sell prices under the MMR, BS, proposed algorithm and the grid import and export prices.

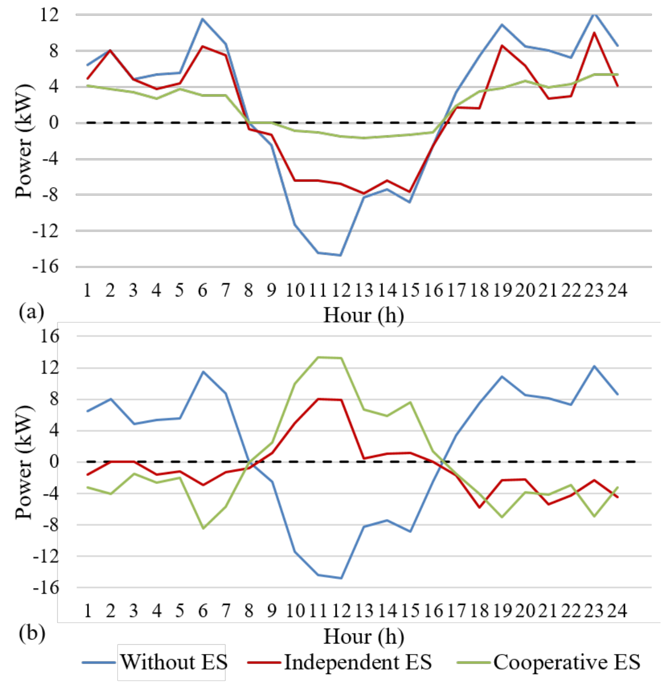

Fig. 4. (a) Net load and generation of the 16-prosumer coalition without ES, with independent ES operation, and with cooperative ES operation; (b) Aggregate ES charging and discharging scheduling with independent and collaborative ES operations.

coalition while Fig. 4 (b) illustrates the aggregate ES charging (positive) and discharging (negative) schedules, for three different cases: i) a case without operating any ES units (blue curves), ii) a case where each prosumer independently manages the operation of its ES unit to minimize its own energy cost (red curves), and iii) a case where prosumers cooperatively manage the operation of their ES units to minimize the total coalitional energy cost (green curves).

It can be observed that under cooperative ES operation, the abundant PV generation during mid-day periods is largely absorbed and stored in the ES and is discharged during periods characterized by high demand and none/low PV production, and the cooperative ES charging and discharging profile almost mirrors the net load profile. This coordinated use of complementary DERs significantly reduces the peak demand and contributes to a much more locally balanced demand and supply compared to independent ES operation. Furthermore, it is evident that under the proposed pricing mechanism, prosumers are properly incentivized to share their surplus PV generation directly with their neighbors, establishing successful P2P energy sharing among prosumers, as opposed to the case under independent ES operations, where prosumers still injecting their excess PV generation to the grid at an unattractive price.

E. Benefit Distribution Stability

The greatest excess represents the *worst-case* excess determined over all possible sub-coalitions (excluding the empty set $\emptyset$) with respect to the benefit distribution solution. As long as the greatest excess is negative, the negativity of excesses of the rest sub-coalitions can be implied. It therefore suffices to analyze only the greatest excess under different mechanisms to study their stability.

Table I illustrates the greatest excess of the proposed and state-of-the-art mechanisms as the number of prosumers N grows larger. We assume that each prosumer has a 50% chance to take the ownership of a PV system or an ES unit. It can be observed that the nucleolus and the proposed algorithm always guarantee the greatest excess to be negative regardless of the number of prosumers, which shows that their benefit distribution solutions are stable and can achieve the sustainable participation of prosumers in P2P trading. On the other hand, the greatest excess is always positive under MMR and BS while can be positive under SV, verifying the statements in Sections III-B, III-C, and III-D that the benefit distribution under these mechanisms may not be in the core of the proposed prosumer coalitional game.

TABLE I
The greatest excess (in pence) under the proposed and state-of-the-art mechanisms for different number of prosumers.

N	4	8	12	16	20
MMR	52.06	85.32	113.43	260.35	304.26
BS	79.68	84.76	158.94	162.41	177.29
SV	20.28	-0.90	11.48	24.97	18.23
Nucleolus	-3.14	-12.70	-5.38	-10.25	-8.89
Proposed	-3.14	-12.70	-5.38	-10.25	-8.89

F. Computational Performance

Table II summarizes the computational performance of the proposed and state-of-the-art mechanisms by presenting the total computational time (and the number of LPs solved to obtain the nucleolus as indicated in parentheses) for an increasing number of prosumers. For a fair comparison, the statistics provided in Table II exclude the computational time of obtaining a complete list of coalition values since such a calculation step is not required under MMR and BS.

TABLE II
Total computational time (in seconds) of the proposed the state-of-the-art mechanisms.

N	4	8	12	16	20
MMR	0.61	0.63	0.62	0.62	0.63
BS	0.58	0.57	0.59	0.58	0.59
SV	0.6	0.63	0.65	0.65	0.66
Nucleolus (Iteration)	2.3 (7)	39.4 (133)	664.33 (1,443)	11,951 (40,006)	94,972 (629,145)
Proposed	0.62	0.69	0.73	0.75	0.79

It can be observed that the total computational time for determining the nucleolus increases exponentially as the number of prosumers increases. As mentioned in Section III-E, this is because the computation of the nucleolus requires the solution of $\mathcal{O}(2^N)$ LPs (e.g. a total number of 629,145 LPs are solved in the case of 20 prosumers). Such massive computational burden largely restricts the adoption of the nucleolus in P2P trading despite that it always belongs to the core. On the other hand, although MMR, BS, and SV are computationally efficient, these mechanisms may not suitably sustain prosumers' participation in P2P energy trading (as demonstrated in Sections V-B and V-E). To this end, the proposed pricing algorithm (which only requires solving a single LP regardless of N) shares the similar computational burden of MMR, BS and SV while always guarantees to incentivize prosumers to stay in the grand coalition.

VI. Conclusion and Future Work

Previously proposed pricing and benefit distribution mechanisms for managing P2P trading may fail to incentivizing sustainable P2P trading among prosumers or is encumbered by significant computational burden. To overcome these challenges, this paper has proposed a P2P energy trading mechanism based on cooperative game theory to construct a grand energy coalition of prosumers and compute the highest energy cost saving by optimizing the operation of prosumers' ES units cooperatively. A computationally efficient pricing algorithm has been proposed to identify a stable distribution of the total cost savings to prosumers and guarantees that none of them can benefit with higher cost savings by exiting the grand coalition to join smaller sub-coalitions. Case studies have been conducted using real-world system data and the results have demonstrated that under the proposed P2P trading and pricing mechanism, the collaborative operation of prosumers' ES units contributes to a more locally balanced demand and supply compared to independent ES operation and enables the excess RES generation to be efficiently shared within the grand coalition. The value of the proposed pricing algorithm has been demonstrated by comparing it against state-of-the-art mechanisms. Results have demonstrated that the proposed algorithm is superior to MMR, BS and SV in financially incentivizing prosumers to stay in the grand coalition and also exhibits a more favorable computational performance than nucleolus.

Future work aims at extending the proposed approach and the presented analysis to incorporate flexible demand technologies in the presented prosumer energy model, such as electric vehicles and smart wet appliances. This will enable a comprehensive analysis of the value of different demand response initiatives in P2P energy trading.

References

[1] A. Shakoor, G. Davies, and G. Strbac, "Roadmap for flexibility services to 2030," *A report to the Committee on Climate Change. London: Pöyry*, May 2017.

[2] M. Khorasany, Y. Mishra, and G. Ledwich, "A decentralised bilateral energy trading system for peer-to-peer electricity markets," *IEEE Trans. Ind. Electron.*, in press.

[3] T. Morstyn, N. Farrell, S. J. Darby, and M. D. McCulloch, "Using peer-to-peer energy-trading platforms to incentivize prosumers to form federated power plants," *Nature Energy*, vol. 3, no. 2, pp. 94–101, Feb. 2018.

[4] O. Jogunola, A. Ikpehai, K. Anoh, B. Adebisi, M. Hammoudeh, S. Y. Son, and G. Harris, "State-of-the-art and prospects for peer-to-peer transaction-based energy system," *Energies*, vol. 10, no. 12, p. 2106, Dec. 2017.

[5] H. Kanchev, F. Colas, V. Lazarov, and B. Francois, "Emission reduction and economical optimization of an urban microgrid operation including dispatched pv-based active generators," *IEEE Trans. Sustain. Energy*, vol. 5, no. 4, pp. 1397–1405, Jul. 2014.

[6] N. Liu, X. Yu, C. Wang, C. Li, L. Ma, and J. Lei, "Energy-sharing model with price-based demand response for microgrids of peer-to-peer prosumers," *IEEE Trans. Power Syst.*, vol. 32, no. 5, pp. 3569–3583, Jan. 2017.

[7] C. Long, J. Wu, C. Zhang, L. Thomas, M. Cheng, and N. Jenkins, "Peer-to-peer energy trading in a community microgrid," in *Proc. 2017 IEEE Power Energy Soc. Gen. Meet.*, Chicago, IL, USA, Jul. 2017, pp. 2–6.

[8] R. Alvaro-Hermana, J. Fraile-Ardanuy, P. J. Zufiria, L. Knapen, and D. Janssens, "Peer to peer energy trading with electric vehicles," *IEEE Intell. Transp. Syst. Mag.*, vol. 8, no. 3, pp. 33–44, Jul. 2016.

[9] C. Zhang, J. Wu, Y. Zhou, M. Cheng, and C. Long, "Peer-to-peer energy trading in a microgrid," *Applied Energy*, vol. 220, pp. 1–12, Jun. 2018.

[10] Y. Liu, L. Wu, and J. Li, "Peer-to-peer (p2p) electricity trading in distribution systems of the future," *The Electricity Journal*, vol. 32, no. 4, pp. 2–6, May 2019.

[11] W. Tushar, T. K. Saha, C. Yuen, P. Liddell, R. Bean, and H. V. Poor, "Peer-to-peer energy trading with sustainable user participation: A game theoretic approach," *IEEE Access*, vol. 6, pp. 62 932–62 943, Oct. 2018.

[12] W. Tushar, T. K. Saha, C. Yuen, T. Morstyn, and M. D. McCulloch, "A motivational game-theoretic approach for peer-to-peer energy trading in the smart grid," *Applied Energy*, vol. 243, pp. 10–20, Jun. 2019.

[13] Y. Zhou, J. Wu, and C. Long, "Evaluation of peer-to-peer energy sharing mechanisms based on a multiagent simulation framework," *Applied Energy*, vol. 222, pp. 993–1022, Jul. 2018.

[14] W. Lee, L. Xiang, R. Schober, and V. W. Wong, "Direct electricity trading in smart grid: A coalitional game analysis," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 7, pp. 1398–1411, Jun. 2014.

[15] L. Han, T. Morstyn, and M. D. McCulloch, "Incentivizing prosumer coalitions with energy management using cooperative game theory," *IEEE Trans. Power Syst.*, vol. 34, no. 1, pp. 303–313, Jul. 2018.

[16] W. Saad, A. L. Glass, N. B. Mandayam, and H. V. Poor, "Toward a consumer-centric grid: A behavioral perspective," *Proc. IEEE*, vol. 104, no. 4, pp. 865–882, Oct. 2016.

[17] Y. Ye, D. Papadaskalopoulos, and G. Strbac, "Investigating the ability of demand shifting to mitigate electricity producers' market power," *IEEE Trans. Power Syst.*, vol. 33, no. 4, pp. 3800–3811, Dec. 2017.

[18] Y. Ye, D. Papadaskalopoulos, R. Moreira, and G. Strbac, "Investigating the impacts of price-taking and price-making energy storage in electricity markets through an equilibrium programming model," *IET Gener. Transm. Distrib.*, vol. 13, no. 2, pp. 305–315, Nov. 2018.

[19] L. S. Shapley, "A value for n-person games," *Contributions to the Theory of Games*, vol. 2, no. 28, pp. 307–317, 1953.

[20] B. Peleg and P. Sudhölter, *Introduction to the theory of cooperative games*. Springer Science & Business Media, 2007.

[21] Ausgrid. (2019) Solar home electricity data. [Online]. Available: https://www.ausgrid.com.au/Industry/Innovation-and-research/Data-to-share/Solar-home-electricity-data

[22] Ofgem. (2018) Uk economy 7. [Online]. Available: https://www.ofgem.gov.uk/ofgem-publications/128730

[23] Ofgem. (2018) Feed-in tariff (fit) rates. [Online]. Available: https://www.ofgem.gov.uk/environmental-programmes/fit/fit-tariff-rates

[24] Fico xpress. [Online]. Available: http://www.fico.com/en/Products/DMTools/Pages/FICO-Xpress-Optimization-Suite.aspx