Affine Policies for Flexibility Provision by Natural Gas Networks to Power Systems

Anubhav Ratha[∗†], Anna Schwele[∗], Jalal Kazempour[∗], Pierre Pinson[∗], Shahab Shariat Torbaghan[†] and Ana Virag[†]
[∗] Center for Electric Power and Energy, Technical University of Denmark, Kgs. Lyngby, Denmark
{arath, schwele, seykaz, ppin}@elektro.dtu.dk
[†] Flemish Institute of Technological Research (VITO), Boeretang 200, 2400 Mol, Belgium
{anubhav.ratha, shahab.shariattorbaghan, ana.virag}@vito.be

***Abstract*—Using flexibility from the coordination of power and natural gas systems helps with the integration of variable renewable energy in power systems. To include this flexibility into the operational decision-making problem, we propose a distributionally robust chance-constrained co-optimization of power and natural gas systems considering flexibility from short-term gas storage in pipelines, i.e., linepack. Recourse actions in both systems, based on linear decision rules, allow adjustments to the dispatch and operating set-points during real-time operation when the uncertainty in wind power production is revealed. We convexify the non-linear and non-convex power and gas flow equations using DC power flow approximation and second-order cone relaxation, respectively. Our coordination approach enables a study of the mitigation of short-term uncertainty propagated from the power system to the gas side. We analyze the results of the proposed approach on a case study and evaluate the solution quality via out-of-sample simulations performed *ex-ante*.**

***Index Terms*—Linear decision rules, Distributionally robust chance constraints, Linepack flexibility, Power and natural gas coordination, Second-order cone program.**

I. Introduction

Natural gas-fired power plants (NGFPPs) typically provide operational flexibility to power systems with a high share of intermittent renewable energy. Short-term gas storage in natural gas pipelines, known as linepack, provides an additional source of flexibility [1] at no extra investment cost. Efficient procurement of flexibility from the natural gas system during day-ahead scheduling of power systems requires consideration of the operational constraints of the natural gas system. Further, with the increasing share of intermittent renewable energy sources in the power system, the need for flexibility and thereby, the interdependence between power and natural gas systems is becoming stronger [2]. As a result, the coordination between power and natural gas systems during the day-ahead dispatch has been a topic of research interest in recent years. For example, various levels of coordination and information exchange between the systems are discussed in [3], [4], while [5], [6], [7], [8], [9] model full integration of the power with the natural gas system. The value of gas system related flexibility for the power system is quantified in [6] and [7] in a deterministic manner.

The work of A. Ratha was supported by a Ph.D. grant provided by the Flemish Institute for Technological Research (VITO) and scholarship from Technical University of Denmark (DTU). The work of A. Schwele and J. Kazempour was supported by the Danish EUD Programme through the Coordinated Operation of Integrated Energy Systems (CORE) project under the grant 64017-0005.

Increasing interactions between power and natural gas systems, however, result in the propagation of short-term uncertainty faced by power systems to the gas side. Prior works on the coordinated operation of power and natural gas systems have largely ignored this short-term uncertainty. This may result in additional recourse actions necessary during the real-time operation stage when the flexibility from the natural gas system is not correctly anticipated. Affine policies, built on the theory of linear decision rules, have been a preferred choice for day-ahead decision making, wherein nominal dispatch schedules along with the recourse actions for real-time operation are optimally decided [10]. In this paper, we introduce a unified framework to elicit flexibility based on affine policies from agents, e.g., power producers, natural gas suppliers as well as the network assets, i.e., linepack. Our affine policies are decided based on the features of uncertainty drawn from the historical measurements, with no distributional restriction imposed on the random variables.

Previous works discussing uncertainty-aware coordination between power and natural gas systems use stochastic programming approaches such as scenario-based [5], robust [8], and chance-constrained optimization [9]. Reference [5] proposes a two-stage stochastic program for the day-ahead and real-time operations of integrated power and natural gas system under uncertainty from renewable generation. In a similar direction, a robust dispatch framework is proposed in [8] which models uncertainty through intervals and extreme scenario approximation. Chance-constraints are introduced into the planning problem of the integrated power and natural gas system [9]. While scenario-based approaches [5] incur a high computational expense due to a large number of scenarios needed to characterize the uncertainty properly, robust approaches [8] often suffer from over-conservativeness of the solution due to the design objective to minimize worst-case cost. Distributionally robust chance-constrained formulation of the problem [11] allows for a tunable probabilistic violation of operational limits when facing extreme realizations of uncertainty which is characterized by an ambiguity set.

In this work, we adopt a distributionally robust chance-constrained optimization technique, considering its advantages over other stochastic programming approaches [11], to introduce a coordinated day-ahead dispatch of power and natural gas systems taking the flexibility provided by linepack into account. To the best of our knowledge, this is the first paper to bring linepack flexibility to the day-ahead dispatch problem, while modeling and mitigating the short-term uncertainty propagated from the power system to the natural gas system. Studying this uncertainty propagation opens new pathways for the endogenous valuation of the natural gas network as a provider of short-term flexibility to power systems. This could potentially result in the design of new market-based coordination mechanisms and market products enabling gas system agents and the network to play an active role in providing flexibility to the power system. From a methodological perspective, our main contribution is a tractable reformulation of distributionally robust chance constraints for the combined power and gas dispatch problem considering linepack.

The rest of this paper is organized as follows: Section II presents the distributionally robust chance-constrained power and natural gas dispatch problem. Section III discusses the solution methodology, which is then applied to a case study in Section IV. Finally, conclusions are drawn and the avenues for future work are discussed in Section V.

II. Problem Formulation

A. Preliminaries

In the following, we introduce the operation of a coupled power and natural gas system, wherein power generated from dispatchable power plants $i \in \mathcal{I}$ and wind farms $j \in \mathcal{J}$ is used to meet the inelastic electricity demand from a set of loads $d \in \mathcal{D}$. The dispatchable generators comprise of NGFPPs $i \in \mathcal{G}$ and non-NGFPPs $i \in \mathcal{C}$, such that $\mathcal{G} \cap \mathcal{C} = \emptyset$ and $\mathcal{G} \cup \mathcal{C} = \mathcal{I}$. On the gas side, natural gas suppliers $k \in \mathcal{K}$, together with available linepack in the gas network, are dispatched to meet the natural gas demand from inelastic gas loads and the fuel needed by NGFPPs. The non-linear and non-convex power and gas flow equations are convexified using DC power flow approximation and second-order cone relaxation, respectively. We assume that wind power is available at zero marginal cost of production. Power produced by wind farms during real-time operation is the sole uncertainty source considered.

B. Uncertainty Model

For wind farm j, the day-ahead point forecast for time period $t \in \mathcal{T}$ is given by $W_{j,t}^{\text{PF}}$. The forecast error observed in real-time is assumed to be a random variable $\delta_{j,t}$, such that the overall system uncertainty can be characterized by $\Omega = [\delta_{11} \ \delta_{21} \ \dots \ \delta_{|\mathcal{J}|t} \ \dots \ \delta_{|\mathcal{J}||\mathcal{T}|}]^\top \in \mathbb{R}^{|\mathcal{J}||\mathcal{T}|}$, where $\mathbb{R}$ is the set of real numbers and $|\cdot|$ is the cardinality operator over a set. We consider that Ω follows an unknown multivariate probability distribution $\mathbb{P} \in \Pi$, where Π is an ambiguity set defined as

$$\Pi = \{\mathbb{P} \in \Pi_0(\mathbb{R}^{|\mathcal{J}|}) : \mathbb{E}_{\mathbb{P}}[\Omega] = \boldsymbol{\mu}^{\Pi}, \mathbb{E}_{\mathbb{P}}[\Omega\Omega^\top] = \boldsymbol{\Sigma}^{\Pi}\}, \quad (1)$$

such that the family of distributions, $\Pi_0(\mathbb{R}^{|\mathcal{J}|})$ contains all probability distributions whose first and second-order moments are given by known parameters $\boldsymbol{\mu}^{\Pi} \in \mathbb{R}^{|\mathcal{J}||\mathcal{T}|}$ and $\boldsymbol{\Sigma}^{\Pi} \in \mathbb{R}^{|\mathcal{J}||\mathcal{T}|\times|\mathcal{J}||\mathcal{T}|}$, respectively. Further, $\mathbb{E}_{\mathbb{P}}[\cdot]$ denotes expectation with respect to the distribution $\mathbb{P}$ and $(\cdot)^\top$ is the transpose operator. Without any loss of generality, we assume that the mean $\boldsymbol{\mu}^{\Pi} = 0$ and that the covariance matrix $\boldsymbol{\Sigma}^{\Pi}$ can be empirically estimated from historical record of wind forecast errors. The structure of the positive semi-definite covariance matrix, $\boldsymbol{\Sigma}^{\Pi}$ is such that its diagonal blocks, comprised of sub-matrices, $\boldsymbol{\Sigma}_t^{\Pi} \in \mathbb{R}^{|\mathcal{J}|\times|\mathcal{J}|}, \forall t \in \mathcal{T}$, capture the spatial correlation among the wind forecast errors in period t, while the off-diagonal blocks contain information about spatio-temporal correlation of the uncertain parameters.

With this description of uncertain wind forecast errors, the net deviation from the point forecasts of all wind farms in the time period t is $\mathbf{e}^\top\Omega_t$ where $\mathbf{e} \in \mathbb{R}^{|\mathcal{J}|}$ is a vector of all ones. The temporally collapsed random vector is formed as: $\Omega_t = F_t\Omega$, where $F_t \in \mathbb{R}^{|\mathcal{J}|\times|\mathcal{J}||\mathcal{T}|}$ is a *selector matrix* formed by blocks of null matrices $\mathbf{0} \in \mathbb{R}^{|\mathcal{J}|\times|\mathcal{J}|}$ and a single block of identity matrix $\mathbf{1} \in \mathbb{R}^{|\mathcal{J}|\times|\mathcal{J}|}$, starting at column $(|\mathcal{J}|(t-1)+1)$, $\forall t \in \mathcal{T}$. As a sign convention, $\mathbf{e}^\top\Omega_t > 0$ implies deficit of wind power available in the system during real-time operation stage as compared to the day-ahead forecast.

C. Uncertainty-Aware Power and Natural Gas Coordination

The proposed day-ahead coordinated electricity and natural gas model is a stochastic program, presented in (2) in the following. The objective function has a min-max structure such that the total *expected* system dispatch cost is minimized while the uncertain variable Ω draws from a probability distribution $\mathbb{P} \in \Pi$ that results in maximizing the expected cost of dispatch, i.e., the *worst-case* probability distribution.

$$\min_{\Theta_1} \max_{\mathbb{P}\in\Pi} \mathbb{E}_{\mathbb{P}}\Bigg[\sum_{t\in\mathcal{T}}\Big(\sum_{i\in\mathcal{C}} C_i^{\text{E}} \ \tilde{p}_{i,t} + \sum_{k\in\mathcal{K}} C_k^{\text{G}} \ \tilde{g}_{k,t}\Big)\Bigg] \quad \text{(2a)}$$

subject to

$$\mathbf{e}^\top\tilde{\mathbf{p}}_t + \mathbf{e}^\top(\mathbf{W}_t^{\text{PF}} - \Omega_t) = \mathbf{e}^\top\mathbf{D}_t^{\text{E}}, \ \forall t, \quad \text{(2b)}$$

$$\tilde{\mathbf{P}}_t^{\text{inj}} = \boldsymbol{\Psi}_{\text{I}}\tilde{\mathbf{p}}_t + \boldsymbol{\Psi}_{\text{J}}(\mathbf{W}_t^{\text{PF}} - \Omega_t) - \boldsymbol{\Psi}_{\text{D}}\mathbf{D}_t^{\text{E}}, \ \forall t, \quad \text{(2c)}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\{\boldsymbol{\Psi}\tilde{\mathbf{P}}_t^{\text{inj}}\}_{(n,r)} \geq -\{\overline{\mathbf{F}}\}_{(n,r)}] \geq (1-\epsilon_{nr}), \ \forall (n,r) \in \mathcal{L}, \ \forall t, \quad \text{(2d)}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\{\boldsymbol{\Psi}\tilde{\mathbf{P}}_t^{\text{inj}}\}_{(n,r)} \leq \{\overline{\mathbf{F}}\}_{(n,r)}] \geq (1-\epsilon_{nr}), \ \forall (n,r) \in \mathcal{L}, \ \forall t, \quad \text{(2e)}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{p}_{i,t} \geq \underline{P}_i] \geq (1-\epsilon_i), \ \forall i, \ \forall t, \quad \text{(2f)}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{p}_{i,t} \leq \overline{P}_i] \geq (1-\epsilon_i), \ \forall i, \ \forall t, \quad \text{(2g)}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{g}_{k,t} \geq \underline{G}_k] \geq (1-\epsilon_k), \ \forall k, \ \forall t, \quad \text{(2h)}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{g}_{k,t} \leq \overline{G}_k] \geq (1-\epsilon_k), \ \forall k, \ \forall t, \quad \text{(2i)}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{pr}_{m,t} \geq \underline{PR}_m] \geq (1-\epsilon_m), \ \forall m, \ \forall t, \quad \text{(2j)}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{pr}_{m,t} \leq \overline{PR}_m] \geq (1-\epsilon_m), \ \forall m, \ \forall t, \quad \text{(2k)}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{pr}_{u,t} \leq \Gamma_{m,u}\ \tilde{pr}_{m,t}] \geq (1-\epsilon_{mu}),$$
$$\forall (m,u) \in \mathcal{Z}_c,\ \forall t, \quad (2l)$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{q}_{m,u,t} \geq 0] \geq (1-\epsilon_{mu}),\ \forall (m,u) \in \mathcal{Z},\ \forall t, \quad (2m)$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{q}^{\text{in}}_{m,u,t} \geq 0] \geq (1-\epsilon_{mu}),\ \forall (m,u) \in \mathcal{Z},\ \forall t, \quad (2n)$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{q}^{\text{out}}_{m,u,t} \geq 0] \geq (1-\epsilon_{mu}),\ \forall (m,u) \in \mathcal{Z},\ \forall t, \quad (2o)$$

$$\tilde{q}^2_{m,u,t} = K^2_{m,u}(\tilde{pr}^2_{m,t} - \tilde{pr}^2_{u,t}),\ \forall (m,u) \in \mathcal{Z},\ \forall t, \quad (2p)$$

$$\tilde{q}_{m,u,t} = \frac{\tilde{q}^{\text{in}}_{m,u,t} + \tilde{q}^{\text{out}}_{m,u,t}}{2},\ \forall (m,u) \in \mathcal{Z},\ \forall t, \quad (2q)$$

$$\tilde{h}_{m,u,t} = S_{m,u} \frac{\tilde{pr}_{m,t} + \tilde{pr}_{u,t}}{2},\ \forall (m,u) \in \mathcal{Z},\ \forall t, \quad (2r)$$

$$\tilde{h}_{m,u,t} = H^0_{m,u} + \tilde{q}^{\text{in}}_{m,u,t} - \tilde{q}^{\text{out}}_{m,u,t}, \forall (m,u) \in \mathcal{Z}, t=1, \quad (2s)$$

$$\tilde{h}_{m,u,t} = \tilde{h}_{m,u,(t-1)} + \tilde{q}^{\text{in}}_{m,u,t} - \tilde{q}^{\text{out}}_{m,u,t},$$
$$\forall (m,u) \in \mathcal{Z}, t>1, \quad (2t)$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{h}_{m,u,t} \geq H^0_{m,u}] \geq (1-\epsilon_{mu}),$$
$$\forall (m,u) \in \mathcal{Z}, t=|\mathcal{T}|, \quad (2u)$$

$$\sum_{k\in\mathcal{A}^{\text{K}}_m} \tilde{g}_{k,t} - \sum_{i\in\mathcal{A}^{\text{G}}_m} \phi_i\ \tilde{p}_{i,t} - \sum_{u:(m,u)\in\mathcal{Z}} (\tilde{q}^{\text{in}}_{m,u,t} - \tilde{q}^{\text{out}}_{u,m,t}) = D^{\text{G}}_{m,t},\ \forall m,\ \forall t, \quad (2v)$$

where the set of stochastic variables is Θ_1={$\tilde{p}_{i,t}$, $\tilde{g}_{k,t}$, $\tilde{pr}_{m,t}$, $\tilde{q}_{m,u,t}$, $\tilde{q}^{\text{in}}_{m,u,t}$, $\tilde{q}^{\text{out}}_{m,u,t}$, $\tilde{h}_{m,u,t}$}. The terms in objective (2a) are the expected cost of power generation by non-NGFPPs and the cost of natural gas supply by gas suppliers derived from marginal production cost C^{E}_i and C^{G}_k, respectively.

The inequalities (2d)-(2o) and (2u) are modeled as distributionally robust chance constraints. This means that at the optimal solution to problem (2), the probability of meeting each individual constraint inside the square brackets $\mathbb{P}[\cdot]$ is modeled to have a confidence level of at least $(1-\epsilon_{(\cdot)})$, where each $\epsilon_{(\cdot)}$ lies within 0 and 1, i.e., $\epsilon_{(\cdot)} \in [0,1]$. Subscripts $(\cdot)$ take the appropriate indices from the set $\{i,\ (n,r),\ k,\ m,\ (m,u)\}$ depending on the individual constraint.

Constraints (2b)-(2g) pertain to the power system. These constraints include the power balance (2b), limits on the stochastic power flows in the transmission lines (2c)-(2e) and the upper ($\underline{P}_i$) and lower bounds ($\overline{P}_i$) on the stochastic power production of generators (2f) and (2g). Vectors $\tilde{\mathbf{p}}_t \in \mathbb{R}^{|\mathcal{I}|}$, $\mathbf{W}^{\text{PF}}_t \in \mathbb{R}^{|\mathcal{J}|}$ and $\mathbf{D}^{\text{E}}_t \in \mathbb{R}^{|\mathcal{D}|}$ represent the power produced by generators, wind forecasts for wind farms and electricity demand from loads in period t, while Ω_t is the random vector of forecast errors, as previously defined. Vector coefficients, $\mathbf{e}$ in (2b) are of appropriate dimensions such that the total supply and demand are balanced in each period t. The matrix $\mathbf{\Psi} \in \mathbb{R}^{|\mathcal{L}|\times|\mathcal{N}|}$ represents the Power Transfer Distribution Factor (PTDF) matrix, derived from the reactances of power transmission lines [12], which maps the injections $\tilde{\mathbf{P}}^{\text{inj}}_t \in \mathbb{R}^{|\mathcal{N}|}$ at the electricity nodes to the power flows in each of the power lines $(n,r) \in \mathcal{L}$ respecting capacity limits $\overline{\mathbf{F}}$ in the network. Similarly, matrices $\mathbf{\Psi}_{\text{I}} \in \mathbb{R}^{|\mathcal{N}|\times|\mathcal{I}|}$, $\mathbf{\Psi}_{\text{J}} \in \mathbb{R}^{|\mathcal{N}|\times|\mathcal{J}|}$, and $\mathbf{\Psi}_{\text{D}} \in \mathbb{R}^{|\mathcal{N}|\times|\mathcal{D}|}$ map generators, wind farms and loads to the electricity nodes, such that (2c) gives the nodal power injections for all electricity nodes in the system.

Natural gas system constraints are given in (2h)-(2u). While constraints (2h) and (2i) limit the stochastic gas supply $\tilde{g}_{k,t}$ by supplier k in time period t to $\underline{G}_k$ and $\overline{G}_k$, (2j) and (2k) limit the nodal gas pressure $\tilde{pr}_{m,t}$ at each gas node $m \in \mathcal{M}$ to be within the physical limits $\underline{PR}_m$ and $\overline{PR}_m$. For the natural gas pipelines with compressors, $\mathcal{Z}_c \subset \mathcal{Z}$, compression is modeled linearly in (2l), which relate the inlet and outlet pressures of two adjacent nodes through compression factor $\Gamma_{m,u}$. We consider that the direction of gas flow in each pipeline $(m,u) \in \mathcal{Z}$ is predetermined and (2m)-(2o) enforce this flow direction in real-time. As remarked in [1], this assumption is non-limiting for the high-pressure, gas transmission networks when considering day-ahead operational problems. On the contrary, it can be a limiting assumption while considering a network expansion planning problem or a gas distribution system wherein injections from distributed gas producers (e.g., biogas plants) cannot be neglected[1]. Equality constraints (2p), known as Weymouth equation, describe the flow $\tilde{q}_{m,u,t}$ (given by (2q) as the average of inflow, $\tilde{q}^{\text{in}}_{m,u,t}$ and outflow, $\tilde{q}^{\text{out}}_{m,u,t}$) along pipeline (m,u) as a quadratic non-convex function of the pressures $\tilde{pr}_{m,t}$ and $\tilde{pr}_{u,t}$ at the inlet (m) and outlet (u) nodes of the pipeline scaled by the pipeline resistance constant $K_{m,u}$. Constraints (2r) define the amount of linepack in the pipelines as the average of inlet and outlet pressures, scaled by the pipeline parameter $S_{m,u}$. Following the modeling approach in [7], (2s)-(2u) describe the evolution of the amount of linepack $\tilde{h}_{m,u,t}$ in a pipeline over time, with (2u) ensuring that the linepack is not depleted at the end of the simulation horizon beyond initial linepack amount $H^0_{m,u}$. Supply-demand balance of natural gas at each node is ensured in real-time by equality constraints (2v) which also couple the power and natural gas systems through the fuel consumed by the NGFPPs scaled by a fuel conversion factor ϕ_i. The sets $\mathcal{A}^{\text{K}}_m \subset \mathcal{K}$ and $\mathcal{A}^{\text{G}}_m \subset \mathcal{G}$ collect gas suppliers and NGFPPs that are located at node m, respectively, while $D^{\text{G}}_{m,t}$ is the nodal gas demand.

The requirement to solve the stochastic program (2) during the day-ahead stage renders the problem infinite dimensional, as the optimization variables are a function of uncertain parameters that are only revealed during real-time operation on the next day. To enable solvability of the problem, we adopt recourse actions based on linear decision rules [13] for the sources of flexibility in the coupled system, i.e., flexible power generation and natural gas supply and linepack. The assumption of affine response to uncertainty by flexible agents, although somewhat limiting in light of the non-linear dynamics of natural gas flow in the network, provides an intuitive understanding of the methodology behind uncertainty propagation from power system to natural gas system at a lower complexity of exposition. Generalized decision rules, for instance as discussed in [14], are left for future work.

[1]The assumptions on fixed gas flow directions may be violated in extreme uncertainty realizations.

D. Affine Policies

When solving the day-ahead dispatch problem, flexible and adjustable agents in the coupled system, i.e., power producers and gas suppliers, are assigned optimal affine policies in addition to the nominal schedule. These affine policies govern their response to the realizations of uncertainty in wind forecast errors during the real-time operation.

a) Power producers: The affine response from dispatchable power plants (NGFPPs and non-NGFPPs) is given by

$$\tilde{p}_{i,t} = p_{i,t} + (\mathbf{e}^\top \Omega_t)\alpha_{i,t},\ \forall i \in \mathcal{I},\ \forall t \quad (3)$$

where $\tilde{p}_{i,t}$ is the stochastic power production of unit i in real-time, $p_{i,t}$ is the nominal power production schedule if the uncertainty were absent (perfect forecasts) and $\alpha_{i,t} \in [-1, 1]$ denotes the *participation factor* of the unit towards mitigation of the deviation.

b) Gas suppliers: The stochastic natural gas supply by supplier k is given by

$$\tilde{g}_{k,t} = g_{k,t} + (\mathbf{e}^\top \Omega_t)\beta_{k,t},\ \forall k \in \mathcal{K},\ \forall t \quad (4)$$

where $g_{k,t}$ is the nominal gas supply and $\beta_{k,t}$ is the participation factor of the supplier towards uncertainty mitigation.

The response to uncertainty by flexible network asset, i.e., linepack, is not directly adjustable as it depends on the allocation of the above affine policies, subject to the topology of the gas network and the physical gas flow constraints.

E. Uncertainty Response by Power and Gas Networks

Here, we discuss how uncertainty affects the flows in the power and gas networks. We consider the case of imperfect forecasts, i.e., $\mathbf{e}^\top \Omega_t \neq 0$.

During the real-time operation, power flows in the transmission lines, modeled by (2c)-(2e) change depending on the realized uncertainty Ω_t, the affine responses of power producers $\alpha_{i,t}$, and the spatial configuration of wind farms and power producers. Moreover, given the response from dispatchable power plants $\alpha_{i,t}$, the power balance constraint (2b) holds true for any realization of uncertainty Ω_t iff

$$\mathbf{e}^\top \mathbf{p}_t + \mathbf{e}^\top \mathbf{W}_t^{\text{PF}} = \mathbf{e}^\top \mathbf{D}_t^{\text{E}},\ \forall t, \quad (5a)$$
$$\mathbf{e}^\top \boldsymbol{\alpha}_t = 1,\ \forall t. \quad (5b)$$

Constraints (5) are derived from (2b) by separating the nominal and uncertainty-dependent terms.

On the gas side, the uncertainty in gas flows, in response to changes in gas supply $\beta_{k,t}$ and in fuel demand from NGFPPs $\phi_i \alpha_{i,t},\ \forall i \in \mathcal{G}$, is mitigated by the flexibility provided by linepack. It is vital to note that the real-time natural gas flows and nodal pressures are functions of $\beta_{k,t}$ and $\alpha_{i,t},\ \forall i \in \mathcal{G}$. However, the analytical derivation of this relationship is not straightforward, given the non-linear gas flow dynamics and the inter-temporal linkages associated with the linepack model. As a simplification, we model the flow and pressure changes as affine functions of uncertainty[2]. We model the real-time natural gas flows in the pipelines as

$$\tilde{q}_{m,u,t} = q_{m,u,t} + (\mathbf{e}^\top \Omega_t)\gamma_{m,u,t},\ \forall (m,u) \in \mathcal{Z},\ \forall t, \quad (6a)$$
$$\tilde{q}^{\text{in}}_{m,u,t} = q^{\text{in}}_{m,u,t} + (\mathbf{e}^\top \Omega_t)\gamma^{\text{in}}_{m,u,t},\ \forall (m,u) \in \mathcal{Z},\ \forall t, \quad (6b)$$
$$\tilde{q}^{\text{out}}_{m,u,t} = q^{\text{out}}_{m,u,t} + (\mathbf{e}^\top \Omega_t)\gamma^{\text{out}}_{m,u,t},\ \forall (m,u) \in \mathcal{Z},\ \forall t, \quad (6c)$$

where $q_{m,u,t},\ q^{\text{in}}_{m,u,t},\ q^{\text{out}}_{m,u,t}$ denote the average flow rate, inflow and outflow rate of natural gas in the pipeline connecting nodes m and u, in absence of forecast errors and the variables $\gamma_{m,u,t},\ \gamma^{\text{in}}_{m,u,t},\ \gamma^{\text{out}}_{m,u,t}$ represent the auxiliary variables which model changes in these flow rates during real-time.

Consequently, the nodal balance constraint for natural gas (2v) holds true for any realization of uncertainty Ω_t iff

$$\sum_{k \in \mathcal{A}^{\text{K}}_m} g_{k,t} - \sum_{i \in \mathcal{A}^{\text{G}}_m} \phi_i\ p_{i,t} - \sum_{u:(m,u) \in \mathcal{Z}} (q^{\text{in}}_{m,u,t} - q^{\text{out}}_{u,m,t}) = D^{\text{G}}_{m,t},\ \forall m,\ \forall t, \quad (7a)$$
$$\sum_{k \in \mathcal{A}^{\text{K}}_m} \beta_{k,t} - \sum_{i \in \mathcal{A}^{\text{G}}_m} \phi_i \alpha_{i,t} - \sum_{u:(m,u) \in \mathcal{Z}} (\gamma^{\text{in}}_{m,u,t} - \gamma^{\text{out}}_{u,m,t}) = 0\ \forall m,\ \forall t. \quad (7b)$$

Constraints (7) are derived by separating the nominal and uncertainty-dependent terms in (2v). Following a similar approach, (2q), $\forall (m,u) \in \mathcal{Z},\ \forall t$, decomposes into

$$q_{m,u,t} = \frac{q^{\text{in}}_{m,u,t} + q^{\text{out}}_{m,u,t}}{2};\ \gamma_{m,u,t} = \frac{\gamma^{\text{in}}_{m,u,t} + \gamma^{\text{out}}_{m,u,t}}{2}. \quad (8)$$

We model real-time pressures at gas nodes as

$$\tilde{pr}_{m,t} = pr_{m,t} + (\mathbf{e}^\top \Omega_t)\rho_{m,t},\ \forall m,\ \forall t, \quad (9)$$

where $pr_{m,t}$ and $\rho_{m,t}$ denote the nominal pressure and the auxiliary variable that models the change in pressure at node m in real-time, respectively. This allows us to expand the Weymouth equation in (2p) as, $\forall (m,u) \in \mathcal{Z},\ \forall t$,

$$\begin{aligned}(q^2_{m,u,t} + (\mathbf{e}^\top \boldsymbol{\Omega}_t)^2\gamma^2_{m,u,t} + 2(\mathbf{e}^\top \boldsymbol{\Omega}_t)\gamma_{m,u,t}q_{m,u,t}) = \\ K^2_{m,u}(pr^2_{m,t} - pr^2_{u,t}) + (\mathbf{e}^\top \boldsymbol{\Omega}_t)^2 K^2_{m,u}(\rho^2_{m,t} - \rho^2_{u,t}) \\ + 2(\mathbf{e}^\top \boldsymbol{\Omega}_t)K^2_{m,u}(\rho_{m,t}pr_{m,t} - \rho_{u,t}pr_{u,t}).\end{aligned} \quad (10)$$

Separating terms that are independent of, quadratically- and linearly-dependent on uncertainty in (10), it can be replaced by the equalities (11) that must hold true for any realization of the uncertainty[3]. For pipelines $\forall (m,u) \in \mathcal{Z},\ \forall t$,

$$q^2_{m,u,t} = K^2_{m,u}(pr^2_{m,t} - pr^2_{u,t}), \quad (11a)$$
$$\gamma^2_{m,u,t} = K^2_{m,u}(\rho^2_{m,t} - \rho^2_{u,t}), \quad (11b)$$
$$\gamma_{m,u,t}q_{m,u,t} = K^2_{m,u}(\rho_{m,t}pr_{m,t} - \rho_{u,t}pr_{u,t}). \quad (11c)$$

[2]In future works, the simplified approach adopted in this paper must be enhanced by considering the true, non-linear analytical relationship of changes in real-time flows and nodal pressures to the affine policies.

[3]Modeling of uncertainty propagation to physical variables such as $\tilde{q}_{m,u,t}$ and $\tilde{pr}_{m,t}$ by estimating sensitivities using Taylor series expansion around the forecast has recently been applied to AC optimal power flow (see, e.g., [15]). Since we solve the dispatch problem in day-ahead, wherein uncertainty around the forecast point is non-negligible, we cannot justify such a sensitivity-based approach.

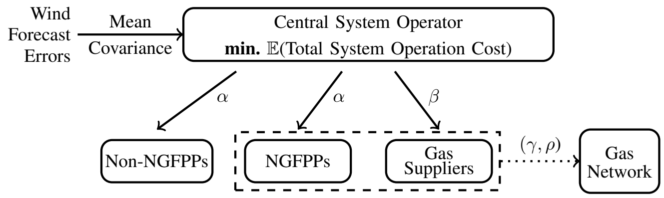


Fig. 1. Coordinated power and natural gas day-ahead dispatch

In addition to the Weymouth equations, the auxiliary variables for flow changes ($\gamma_{m,u,t},\ \gamma^{\text{in}}_{m,u,t},\ \gamma^{\text{out}}_{m,u,t}$) and pressure changes ($\rho_{m,t}$) are coupled by the equality constraints (2r)-(2t) that govern the amount of linepack and evolution of linepack in the pipelines. On separating nominal and uncertainty-dependent terms, these constraints should hold true for any realization of Ω_t iff the equalities (12) hold. For pipelines $\forall(m,u) \in \mathcal{Z}$,

$$h_{m,u,t} = H^0_{m,u} + q^{\text{in}}_{m,u,t} - q^{\text{out}}_{m,u,t},\ \ t = 1, \tag{12a}$$

$$h_{m,u,t} = h_{m,u,(t-1)} + (q^{\text{in}}_{m,u,t} - q^{\text{out}}_{m,u,t}),\ \ t > 1, \tag{12b}$$

$$\frac{S_{m,u}}{2}(\rho_{m,t} + \rho_{u,t} - \rho_{m,(t-1)} - \rho_{u,(t-1)}) = (\gamma^{\text{in}}_{m,u,t} - \gamma^{\text{out}}_{m,u,t}),\ \ t > 1, \tag{12c}$$

where $h_{m,u,t}$ is the nominal linepack in the pipeline in case perfect forecasts of wind power production were to be realized. It is worth noting that, considering the initial linepack amount $H^0_{m,u}$ is uncertainty-independent, constraint (2s) decomposes solely as (12a). Whereas the linepack amount in hours $t > 1$, given by (2t), decomposes as nominal (12b) and uncertainty-dependent (12c) equalities, which govern the change in nominal linepack amount and the response to uncertainty during real-time operation, respectively.

F. Power and Natural Gas Coordination with Affine Policies

In the following we present a finite-dimensional solvable approximation of the stochastic program (2), under the strategy of affine response to uncertainty. As shown in Fig. 1, this day-ahead problem is solved by a central system operator.

$$\min_{\Theta_2} \sum_{t\in\mathcal{T}} \Big(\sum_{i\in\mathcal{C}} C^{\text{E}}_i\ p_{i,t} + \sum_{k\in\mathcal{K}} C^{\text{G}}_k\ g_{k,t} \Big) \tag{13a}$$

subject to

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[p_{i,t} + (\mathbf{e}^\top\Omega_t)\alpha_{i,t} \geq \underline{P}_i] \geq (1-\epsilon_i),\ \ \forall i,\ \forall t, \tag{13b}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[p_{i,t} + (\mathbf{e}^\top\Omega_t)\alpha_{i,t} \leq \overline{P}_i] \geq (1-\epsilon_i),\ \ \forall i,\ \forall t, \tag{13c}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[g_{k,t} + (\mathbf{e}^\top\Omega_t)\beta_{k,t} \geq \underline{G}_k] \geq (1-\epsilon_k),\ \ \forall k,\ \forall t, \tag{13d}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[g_{k,t} + (\mathbf{e}^\top\Omega_t)\beta_{k,t} \leq \overline{G}_k] \geq (1-\epsilon_k),\ \ \forall k,\ \forall t, \tag{13e}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[pr_{m,t} + (\mathbf{e}^\top\Omega_t)\rho_{m,t} \geq \underline{PR}_m] \geq (1-\epsilon_m), \forall m, \forall t, \tag{13f}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[pr_{m,t} + (\mathbf{e}^\top\Omega_t)\rho_{m,t} \leq \overline{PR}_m] \geq (1-\epsilon_m),\ \ \forall m,\ \forall t, \tag{13g}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[(pr_{u,t} - \Gamma_{m,u}\ pr_{m,t}) + (\mathbf{e}^\top\Omega_t)(\rho_{u,t} \tag{13h}$$

$$- \Gamma_{m,u}\ \rho_{m,t}) \leq 0] \geq (1-\epsilon_{mu}),\ \ \forall(m,u) \in \mathcal{Z}_c,\ \forall t, \tag{13i}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[q_{m,u,t} + (\mathbf{e}^\top\Omega_t)\gamma_{m,u,t} \geq 0] \geq (1-\epsilon_{mu}),\ \ \forall(m,u) \in \mathcal{Z},\ \forall t, \tag{13j}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[q^{\text{in}}_{m,u,t} + (\mathbf{e}^\top\Omega_t)\gamma^{\text{in}}_{m,u,t} \geq 0] \geq (1-\epsilon_{mu}),\ \ \forall(m,u) \in \mathcal{Z},\ \forall t, \tag{13k}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[q^{\text{out}}_{m,u,t} + (\mathbf{e}^\top\Omega_t)\gamma^{\text{out}}_{m,u,t} \geq 0] \geq (1-\epsilon_{mu}),\ \ \forall(m,u) \in \mathcal{Z},\ \forall t, \tag{13l}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[h_{m,u,t} + S_{m,u}(\mathbf{e}^\top\Omega_t)\frac{\rho_{m,t} + \rho_{u,t}}{2} \geq H^0_{m,u}] \geq (1-\epsilon_{mu}),\ \ \forall(m,u) \in \mathcal{Z},\ t = |\mathcal{T}|, \tag{13m}$$

$$\text{(2c) - (2e), (5), (7), (8), (11), (12),} \tag{13n}$$

where the optimization variables are $\Theta_2 = \{\ p_{i,t},\ \alpha_{i,t},\ g_{k,t},\ \beta_{k,t},\ pr_{m,t},\ \rho_{m,t},\ q_{m,u,t},\ \gamma_{m,u,t},\ q^{\text{in}}_{m,u,t},\ \gamma^{\text{in}}_{m,u,t},\ q^{\text{out}}_{m,u,t},\ \gamma^{\text{out}}_{m,u,t},\ h_{m,u,t}\}$. The expectation term in objective (2a) reduces to (13a) on account of the zero-mean assumption of Ω_t. As discussed in [16] for programs with a similar structure, the stochastic program (13) is computationally intractable due to the probabilistic distributionally robust chance constraints. To achieve tractability, a convex second-order cone (SOC) approximation of the non-convex individual distributionally robust chance constraints is adopted. Furthermore, the non-convex quadratic equality constraints (11) representing the Weymouth equation for the uncertainty-aware gas flows require convexification. The approach towards solving (13) along with its final tractable form is discussed in the next section.

III. Solution Approach

A. SOC Reformulation of Probabilistic Constraints

For distributionally robust individual chance constraints under the assumption of known first and second-order moments of the underlying probability distribution, [17, Theorem 2.2] provides a SOC approximation based on a variant of Chebyshev's Inequality. While interested readers are directed to [17] for a proof, convex reformulation of constraint (13c) is presented below as an illustration.

With $\mathbf{\Sigma}^\Pi_t \in \mathbb{R}^{|\mathcal{J}|\times|\mathcal{J}|}$ as the t-th diagonal sub-matrix of the covariance matrix $\mathbf{\Sigma}^\Pi$ in time period t and $\mathbf{e} \in \mathbb{R}^{|\mathcal{J}|}$ denoting a vector of all ones, the probabilistic chance constraints (13c) can be approximated by the following SOC constraints:

$$\sqrt{\frac{1-\epsilon_i}{\epsilon_i}}\Big\|\alpha_{i,t}\mathbf{e}^\top(\mathbf{\Sigma}^\Pi_t)^{1/2}\Big\|_2 \leq -p_{i,t} + \overline{P}_i,\ \ \forall i,\ \forall t. \tag{14}$$

Similar reformulation is performed for the other distributionally robust chance constraints in (13). References [18] and [19] remark that such conic reformulation based on Chebyshev's Inequality results in over-conservative solutions as $\epsilon_i \to 0$ while approaching infeasibility for $\epsilon_i \approx 0$. Exact reformulation of such chance constraints improving on this issue has been recently proposed in [18]. However, since the focus of this work is on uncertainty-aware coordination between electricity and natural gas systems, our formulation is limited to the conic approximation. We ensure that large enough risk measures $\epsilon_{(\cdot)}$ are considered in the case study (Section IV) such that infeasibility is avoided.

TABLE I
VARIABLE BOUNDS FOR MCCORMICK RELAXATION.

Variable	Lower bound	Upper bound
$pr_{m,t}$	$\underline{PR}_m$	$\overline{PR}_m$
$\rho_{m,t}$	$-(\overline{PR}_m - \underline{PR}_m)/\widehat{W}$	$(\overline{PR}_m - \underline{PR}_m)/\widehat{W}$
$q_{m,u,t}$	0	$\overline{Q}$
$\gamma_{m,u,t}$	$-\overline{Q}/\widehat{W}$	$\overline{Q}/\widehat{W}$

B. Convex Relaxation of Weymouth Equation

The non-convex quadratic equality constraints in (11a) can be equivalently written as

$$q^2_{m,u,t} \leq K^2_{m,u}(pr^2_{m,t} - pr^2_{u,t}),\ \ \forall(m,u) \in \mathcal{Z},\ \forall t, \tag{15a}$$
$$q^2_{m,u,t} \geq K^2_{m,u}(pr^2_{m,t} - pr^2_{u,t}),\ \ \forall(m,u) \in \mathcal{Z},\ \forall t. \tag{15b}$$

To relax (11a), we adopt the convex SOC constraints (15a) and drop the non-convex constraints (15b)[4]. The tightness of this relaxation is analyzed in [20] and will be further examined in Section IV. Note that (11b) can be convexified in the same manner. However, this convexification strategy cannot be applied to (11c). We adopt McCormick relaxation [21], defining rectangular envelopes around the bi-linear terms in (11c) based on the known and estimated bounds on variables. We first define auxiliary variables $\nu_{m,t}$ for gas nodes $m \in \mathcal{M}$ and $\lambda_{m,u,t}$ for the pipelines $(m,u) \in \mathcal{Z},\ \forall t$ and then replace (11c) by the following set of constraints:

$$\lambda_{m,u,t} - K^2_{m,u}\nu_{m,t} + K^2_{m,u}\nu_{u,t} = 0,\ \forall(m,u) \in \mathcal{Z},\ \forall t, \tag{16a}$$
$$\lambda_{m,u,t} = q_{m,u,t}\gamma_{m,u,t}, \forall(m,u) \in \mathcal{Z}, \forall t, \tag{16b}$$
$$\nu_{m,t} = pr_{m,t}\rho_{m,t},\ \forall m,\ \forall t, \tag{16c}$$
$$\nu_{u,t} = pr_{u,t}\rho_{u,t},\ \forall u : (m,u) \in \mathcal{Z},\ \forall t. \tag{16d}$$

To illustrate the McCormick relaxation, the inequalities that replace the non-convex constraints (16c) are

$$\forall m, t \begin{cases} \rho^L_{m,t}pr_{m,t} + pr^L_{m,t}\rho_{m,t} \leq \nu_{m,t} + pr^L_{m,t}\rho^L_{m,t} \\ \rho^U_{m,t}pr_{m,t} + pr^U_{m,t}\rho_{m,t} \leq \nu_{m,t} + pr^U_{m,t}\rho^U_{m,t} \\ \rho^L_{m,t}pr_{m,t} + pr^U_{m,t}\rho_{m,t} \geq \nu_{m,t} + pr^U_{m,t}\rho^L_{m,t} \\ \rho^U_{m,t}pr_{m,t} + pr^L_{m,t}\rho_{m,t} \geq \nu_{m,t} + pr^L_{m,t}\rho^U_{m,t}, \end{cases} \tag{17}$$

where superscripts L and U indicate lower and upper bounds of the variables, respectively. Constraints (16b) and (16d) are treated similarly. The variable bounds used to construct the McCormick envelopes are listed in Table I. Parameter $\widehat{W}$ is the total installed wind capacity in the system and parameter $\overline{Q}$ denotes the upper bound on gas flow in the pipelines, which we obtain by solving a deterministic version of problem (2) with $\mathbf{e}^\top\Omega_t = 0$. The bounds for network response variables, $\gamma_{m,u,t}$ and $\rho_{m,u,t}$ are trivially deduced from equations (6a) and (9), respectively.

Following the convex approximation of probabilistic constraints and relaxation of Weymouth equations, the tractable form of the distributionally robust chance-constrained day-ahead coordinated power and natural gas dispatch is presented in Appendix A. Problem (20) is a convex second-order cone program (SOCP) and is solvable using off-the-shelf convex optimization solvers.

[4]Linear approximations of the dropped non-convex constraints, as proposed by [1] in a deterministic setting, may also be included to the problem.

IV. CASE STUDY

A. Input Data

A coupled power and natural gas system consisting of a 12-node gas network connected to the IEEE 24-bus reliability test system [5] is used to evaluate our proposed coordinated dispatch. The installed wind capacity reaches one-thirds of the peak demand in the simulation horizon of 24 hours. Data for the parameters of the power and natural gas networks and for the operational characteristics of all assets in the system are provided in online appendix [22]. A dataset of 1,000 zero-mean wind forecast error scenarios based on actual measurements recorded in Western Denmark [23] is used to empirically estimate the covariance matrix $\mathbf{\Sigma}^\Pi$. The parameters $\epsilon_{(\cdot)}$ for all distributionally robust chance constraints in (20) are set to identical values.

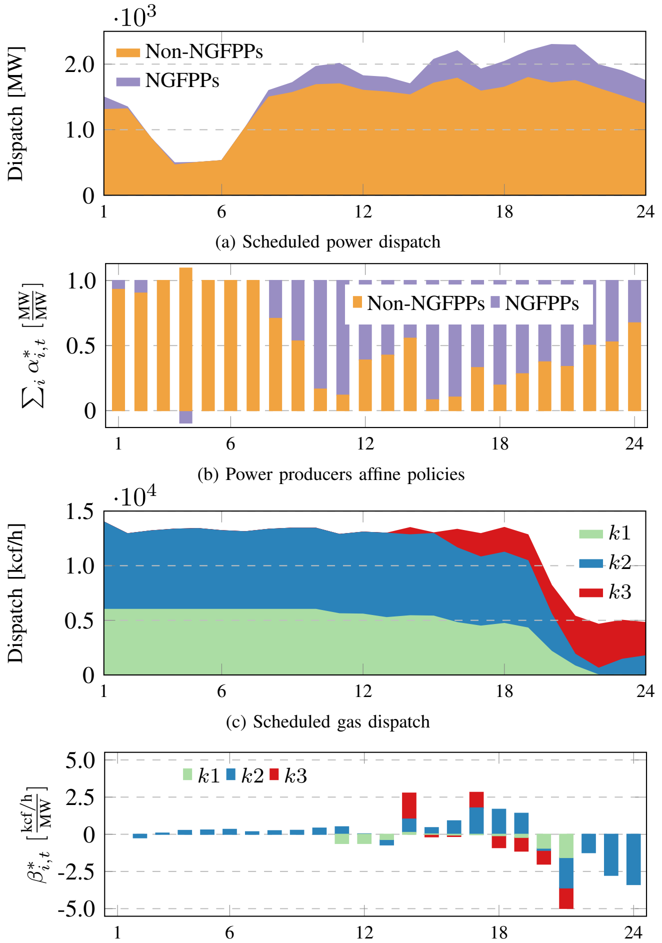
The problem is implemented in Julia v1.1.1 modeled with JuMP v0.2 and solved to optimality by Mosek v9.0 with an average CPU time of 1.67 seconds on a personal computer with 8GB memory running on Intel Core i5 clocked at 2.3 GHz. The optimal solution provides nominal dispatch schedule as well as affine policies that quantify the response to uncertain wind realizations during real-time.

B. Optimal Affine Policies

In Fig. 2, we show the optimal allocations from the distributionally robust chance-constrained day-ahead coordinated power and natural gas model (20) for violation probabilities $\epsilon_{(\cdot)}$ set to 0.05. Fig. 2(a) shows the nominal dispatch of NGFPPs and non-NGFPPS to meet the forecasted net electricity demand, i.e., load minus wind production forecast, while Fig. 2(b) shows their affine responses to uncertainty. Similarly, Figs. 2(c)-(d) present the nominal schedule and the response policies for the three gas suppliers. We highlight our main observations in the following.

First, when power producers and gas suppliers are either dispatched not at all or at full capacity, they are not eligible to adjust their output to mitigate uncertainty. Thus, the response policies for these units are zero. As a result, expensive generators, which are not dispatched in hours with low net demand, are assigned zero α_i in these hours. Similarly, the most expensive gas supplier ($k3$) is not expected to respond to uncertainty in hours 1-13, while not dispatched. On the other hand, the least expensive gas supplier ($k1$) cannot provide a response to uncertainty in hours 1-10, because her nominal dispatch is already at maximum capacity.

Second, NGFPPs are the main providers of flexibility in response to wind uncertainty during hours 8-24, see Fig. 2(b). Although the volatility of gas demand from NGFPPs can be mitigated by linepack, gas suppliers are also required to respond to uncertainty, especially in hours 14-24, see Fig. 2(d).



(a) Scheduled power dispatch

(b) Power producers affine policies

(c) Scheduled gas dispatch

(d) Gas suppliers affine policies

Fig. 2. Optimal dispatch and affine policies for $\epsilon_{(\cdot)} = 0.05$ for the simulation horizon of 24 hours.

Not only the availability and cost structure of power and gas supply, but also network effects impact the optimal response. The spatial correlations of uncertain wind forecasts and location of flexibility providers in both power and gas networks affect the response policies. An example of network effects is the allocation of affine policies in hour 4 in Fig. 2(b). Here, flexibility is provided not only with respect to cost efficiency, but also considering locational benefits and preferable energy flow effects.

C. *Choice of Violation Probabilities* $\epsilon_{(\cdot)}$

To evaluate the quality of the solution obtained and to make an informed choice for $\epsilon_{(\cdot)}$, we perform *ex-ante* simulations using a test dataset of wind realization scenarios, distinct from those used to estimate the covariance matrix. With fixed day-ahead decisions, i.e., nominal production schedules and affine policies, we compute the violation probability of the distributionally robust chance constraints (13b)-(13m) and (2d)-(2e) for a choice of $\epsilon_{(\cdot)}$ as

$$\eta_{\epsilon} = \frac{1}{N_s} \sum_{s=1}^{N_s} \mathbb{I}_s. \tag{18}$$

The indicator function $\mathbb{I}_s$ takes a value 1 if at least one of these

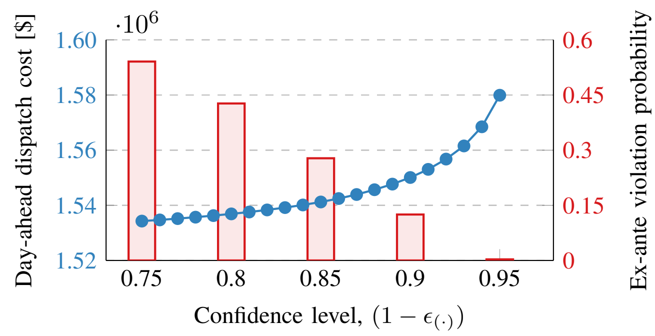


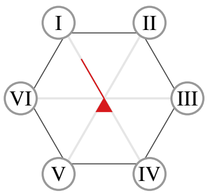
Fig. 3. Day-ahead dispatch cost (left y-axis) with values of $\epsilon_{(\cdot)}$ chosen for the distributionally robust chance constraints is shown by line with markers ●. The ex-ante violation probability (right y-axis) of these constraints, evaluated for 1,000 test samples, is shown in bars.

constraints is violated for the wind realization that corresponds to scenario s. Referring to the left-hand y-axis, the lineplot in Fig. 3 shows the expected cost of day-ahead dispatch at various values of confidence levels $(1 - \epsilon_{(\cdot)})$ imposed on the probabilistic constraints. With a higher confidence of meeting the constraints, the expected cost of day-ahead dispatch increases. The bars, which refer to the right-hand y-axis, show the ex-ante violation probability computed at selected confidence level values. For $\epsilon_{(\cdot)} = 0.05$, an ex-ante violation probability of 0.003 is expected at a day-ahead expected dispatch cost of \$1,580,000.

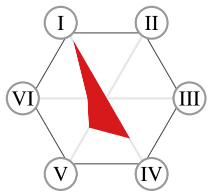
D. *Ex-Ante Violation Probabilities*

Next, we analyse the violation probabilities (18) for each of the following chance-constraints: I. generator bounds (13b) and (13c), II. line flow limits (2d) and (2e), III. non-depletion of linepack in pipelines requirement (13m), IV. natural gas flow direction constraints (13j)-(13l), V. nodal gas pressure bounds (13f) and (13g), and VI. gas supplier bounds (13d) and (13e). Fig. 4 shows the probability of violation of these individual constraints for different choices of $\epsilon_{(\cdot)}$.

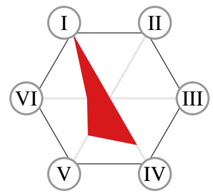
Power generation limits (13b) and (13c) are most susceptible to violation at all values of $\epsilon_{(\cdot)}$. Power transmission lines are not prone to reaching their operational limits. We do not observe any violation probability of power flow limits until decreasing the confidence level to 0.75. On the gas side, the constraints of gas flow directions (13j)-(13l) are susceptible to violations, causing the dependent nodal pressure limits (13f) and (13g) to be violated as well. On the one hand, this can be explained by the relaxation gap for the gas flow equations, which is discussed in detail in the following. On the other hand, this motivates future work to consider bi-directional gas flows, specially in the context of flexibility provision by linepack, albeit at the cost of losing convexity due to introduction of integer variables. The non-depletion of linepack constraints are, however, satisfied even at $\epsilon_{(\cdot)} = 0.25$. This indicates that there is enough short-term gas storage available in the gas pipelines such that they are not depleted at the end of the day while providing flexibility to the power system. Notably, these outcomes and resulting inferences are system-specific.



(a) $\epsilon_{(\cdot)} = 0.05$

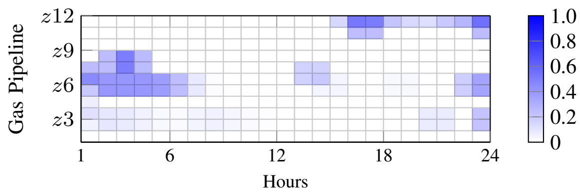


(b) $\epsilon_{(\cdot)} = 0.10$

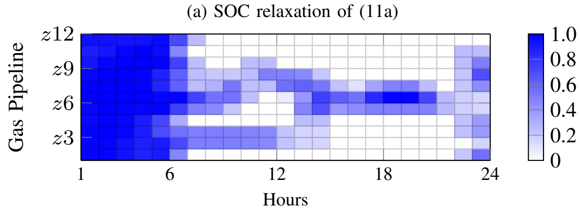


(c) $\epsilon_{(\cdot)} = 0.15$

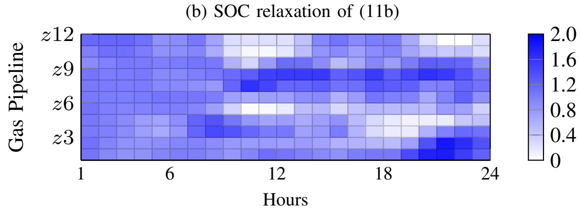
Fig. 4. Ex-ante violation probabilities by constraint type. Corners of the hexagons represent a violation probability of 0.005, 0.15 and 0.3 for (a), (b) and (c), respectively.



(a) SOC relaxation of (11a)



(b) SOC relaxation of (11b)



(c) McCormick Relaxation of (11c)

Fig. 5. Normalized relaxation gap for the convex relaxations adopted for Weymouth equation in (11) for $\epsilon_{(\cdot)} = 0.05$.

E. Tightness of Convex Relaxations

We examine the tightness of the relaxation for the non-convex Weymouth equation (11) by comparing the left-hand and right-hand sides of each of these equality constraints. We define the normalized root mean square relaxation gap Ξ for the original equality constraint $X_{m,u,t} = Y_{m,u,t}, \forall (m,u) \in \mathcal{Z},\ \forall t$ relaxed to $X_{m,u,t} \leq Y_{m,u,t}, \forall (m,u) \in \mathcal{Z},\ \forall t$ as

$$\Xi = \left[\frac{1}{|\mathcal{Z}||\mathcal{T}|} \sum_{t \in \mathcal{T}} \sum_{z \in \mathcal{Z}} \left(\frac{Y^*_{m,u,t} - X^*_{m,u,t}}{Y^*_{m,u,t}} \right)^2 \right]^{\frac{1}{2}}, \qquad (19)$$

where superscript $*$ indicates values obtained at optimality. For $\epsilon_{(\cdot)} = 0.05$, we observe a Ξ-value of 0.78, 1.67 and 2.87 for (11a), (11b) and (11c), respectively.

Fig. 5 presents heatmaps of the normalized root mean square relaxation gap Ξ for each gas pipeline in each hour of the simulation horizon for $\epsilon_{(\cdot)} = 0.05$. The occurrence of relaxation gap is lower for constraint (11a) than for (11b). While the relaxation of constraint (11a) seems to be sufficiently tight in Fig. 5(a), the relaxation of (11b) is not always exact, see Fig. 5(b). The relaxation gap is particularly extant in hours 1-6. The structure of the gas network, which is non-radial and cyclical, and the inter-temporal dynamics of linepack contribute to the lack of tightness of the relaxations. Conditions for exactness of the relaxation of the Weymouth equation can be found in [20] and [24], while approaches for tightening these SOC relaxations are proposed in [1] and [25]. For the McCormick relaxation of constraint (11c) the relaxation gap occurs very frequently and with high severity, see Fig. 5(c). Adversely negative values of γ and/or ρ in the bilinear terms lead to normalized relaxation gaps even larger than 1. Improvements on this approach, such as iterative tightening of the bounds or by convex quadratic enhancement of McCormick relaxation as proposed in [26], will be considered in future works. However, in the context of the proposed coordinated day-ahead dispatch, the tightness of the relaxation is of limited importance, since an additional gas flow feasibility problem [24] is expected to be solved closer to real-time by the gas network operator.

V. Conclusion and Future Perspectives

A. Conclusion

We proposed a distributionally robust chance-constrained coordination of power and natural gas systems to study the propagation of uncertainty from the power to the gas side. Our tractable reformulation of the stochastic program, using recourse actions from the flexible agents in the coupled system and adopting a simplified model for real-time gas flows and nodal pressures, results in a convex SOCP. Ex-ante out-of-sample evaluations are used to demonstrate the quality of the solution while highlighting a trade-off between dispatch cost and violation probability, which influences the choice of allowable violation probabilities. The proposed coordination model enables efficient harnessing of short-term flexibility from the assets in natural gas networks for power systems facing uncertainty. Analysis of the optimal affine policies highlights that our proposed approach enables cost-efficient dispatch and allocation of flexibility across energy sectors facing spatio-temporal effects of uncertainty.

B. Future Perspectives

For future works, detailed out-of-sample simulation studies should be undertaken to better understand the quality of optimal affine responses. Studying the impact of the response policies on the feasibility of the physical constraints of power and natural gas networks in real-time operation and testing the severity of allowed constraint violations are of interest. Convexity-preserving algorithms that tighten the relaxation of gas flow equations can be employed in future works. Further, power-to-gas units that provide additional inter-sectoral flexibility could be included in the model.

Analyzing the proposed coordination in a market context wherein payments for the provision of flexibility-as-a-service are considered, is an interesting topic to investigate in future. Moreover, the impact of limited information sharing among sectors as opposed to the central dispatch considered in this work would be highly insightful. Finally, a market clearing mechanism involving auctions that elicit flexibility from the natural gas sector is a viable pathway towards real-world implementation that is opened up by this paper.

APPENDIX A

The final tractable form of the proposed distributionally robust chance-constrained coordination of power and natural gas systems is the SOCP presented below:

$$\min_{\Theta_2 \cup \{\lambda_{m,u,t},\ \nu_{m,t}\}} \sum_{t\in\mathcal{T}} \Big(\sum_{i\in\mathcal{C}} C_i^{\mathrm{E}}\ p_{i,t} + \sum_{k\in\mathcal{K}} C_k^{\mathrm{G}}\ g_{k,t} \Big) \quad (20a)$$

subject to

$$\xi_i \left\| -\alpha_{i,t} \mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2 \le p_{i,t} - \underline{P}_i,\ \ \forall i,\ \forall t, \quad (20b)$$

$$\xi_i \left\| \alpha_{i,t} \mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2 \le -p_{i,t} + \overline{P}_i,\ \ \forall i,\ \forall t, \quad (20c)$$

$$\xi_{nr} \left\| \{\mathbf{\Psi}(\mathbf{\Psi}_{\mathrm{I}}\boldsymbol{\alpha}_t\mathbf{e}^\top - \mathbf{\Psi}_{\mathrm{J}})\}_{(n,r)} (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2 \le \{\overline{\mathbf{F}}$$
$$+\mathbf{\Psi}(\mathbf{\Psi}_{\mathrm{D}}\mathbf{D}_t^{\mathrm{E}} - \mathbf{\Psi}_{\mathrm{I}}\mathbf{p}_t - \mathbf{\Psi}_{\mathrm{J}}\mathbf{W}_t^{\mathrm{PF}})\}_{(n,r)},\ \ \forall (n,r)\in\mathcal{L},\ \forall t, \quad (20d)$$

$$\xi_{nr} \left\| -\{\mathbf{\Psi}(\mathbf{\Psi}_{\mathrm{I}}\boldsymbol{\alpha}_t\mathbf{e}^\top - \mathbf{\Psi}_{\mathrm{J}})\}_{(n,r)} (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2 \le \{\overline{\mathbf{F}}$$
$$-\mathbf{\Psi}(\mathbf{\Psi}_{\mathrm{D}}\mathbf{D}_t^{\mathrm{E}} - \mathbf{\Psi}_{\mathrm{I}}\mathbf{p}_t - \mathbf{\Psi}_{\mathrm{J}}\mathbf{W}_t^{\mathrm{PF}})\}_{(n,r)},\ \ \forall (n,r)\in\mathcal{L},\ \forall t, \quad (20e)$$

$$\xi_k \left\| -\beta_{k,t} \mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2 \le g_{k,t} - \underline{G}_i,\ \ \forall k,\ \forall t, \quad (20f)$$

$$\xi_k \left\| \beta_{k,t} \mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2 \le -g_{k,t} + \overline{G}_i,\ \ \forall k,\ \forall t, \quad (20g)$$

$$\xi_m \left\| -\rho_{m,t} \mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2 \le pr_{m,t} - \underline{PR}_m,\ \ \forall m,\ \forall t, \quad (20h)$$

$$\xi_m \left\| \rho_{m,t} \mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2 \le -pr_{m,t} + \overline{PR}_m,\ \ \forall m,\ \forall t, \quad (20i)$$

$$\xi_{mu} \left\| (\rho_{u,t} - \Gamma_{m,u}\rho_{m,t}) \mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2$$
$$\le \Gamma_{m,u} pr_{m,t} - pr_{u,t},\ \ \forall (m,u)\in\mathcal{Z}_c,\ \forall t, \quad (20j)$$

$$\xi_{mu} \left\| -\gamma_{m,u,t} \mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2 \le q_{m,u,t}, \forall (m,u)\in\mathcal{Z}, \forall t, \quad (20k)$$

$$\xi_{mu} \left\| -\gamma_{m,u,t}^{\mathrm{in}} \mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2 \le q_{m,u,t}^{\mathrm{in}}, \forall (m,u)\in\mathcal{Z}, \forall t, \quad (20l)$$

$$\xi_{mu} \left\| -\gamma_{m,u,t}^{\mathrm{out}} \mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2 \le q_{m,u,t}^{\mathrm{out}}, \forall (m,u)\in\mathcal{Z}, \forall t, \quad (20m)$$

$$\gamma_{m,u,t}^2 \le K_{m,u}^2(\rho_{m,t}^2 - \rho_{u,t}^2),\ \ \forall (m,u)\in\mathcal{Z},\ \forall t, \quad (20n)$$

$$\xi_{mu} \left\| -(\rho_{m,t} + \rho_{u,t})(\frac{S_{m,u}}{2})^{\frac{1}{2}} \mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \right\|_2$$
$$\le h_{m,u,t} - H_{m,u}^0,\ \ \forall (m,u)\in\mathcal{Z},\ t = |\mathcal{T}|, \quad (20o)$$

McCormick envelopes of (16b) and (16d), (20p)

(12), (7), (5), (15a), (16a), (17), (20q)

where $\xi_i = \sqrt{\frac{1-\epsilon_i}{\epsilon_i}}$, $\xi_{nr} = \sqrt{\frac{1-\epsilon_{nr}}{\epsilon_{nr}}}$, $\xi_k = \sqrt{\frac{1-\epsilon_k}{\epsilon_k}}$, $\xi_m = \sqrt{\frac{1-\epsilon_m}{\epsilon_m}}$, $\xi_{mu} = \sqrt{\frac{1-\epsilon_{mu}}{\epsilon_{mu}}}$ are parameters.

REFERENCES

[1] S. Chen, A. J. Conejo, R. Sioshansi, and Z. Wei, "Unit commitment with an enhanced natural gas-flow model," *IEEE Transactions on Power Systems*, vol. 34, no. 5, pp. 3729–3738, 2019.

[2] P. Pinson, L. Mitridati, C. Ordoudis, and J. Østergaard, "Towards fully renewable energy systems: Experience and trends in Denmark," *CSEE Journal of Power and Energy Systems*, vol. 3, no. 1, pp. 26–35, 2017.

[3] G. Byeon and P. Van Hentenryck, "Unit commitment with gas network awareness," *IEEE Transactions on Power Systems*, vol. 35, no. 2, pp. 1327–1339, 2020.

[4] A. Zlotnik, L. Roald, S. Backhaus, M. Chertkov, and G. Andersson, "Coordinated scheduling for interdependent electric power and natural gas infrastructures," *IEEE Transactions on Power Systems*, vol. 32, no. 1, pp. 600–610, 2017.

[5] C. Ordoudis, P. Pinson, and J. M. Morales, "An integrated market for electricity and natural gas systems with stochastic power producers," *European Journal of Operational Research*, vol. 272, no. 2, pp. 642–654, 2019.

[6] H. Ameli, M. Qadrdan, and G. Strbac, "Coordinated operation strategies for natural gas and power systems in presence of gas-related flexibilities," *Energy Systems Integration*, vol. 1, no. 1, pp. 3–13, 2019.

[7] A. Schwele, C. Ordoudis, J. Kazempour, and P. Pinson, "Coordination of power and natural gas systems: Convexification approaches for linepack modeling," in *Proc. 13th IEEE PES PowerTech Conference*, Milan, 2019.

[8] C. Wang, W. Wei, J. Wang, and T. Bi, "Convex optimization based adjustable robust dispatch for integrated electric-gas systems considering gas delivery priority," *Applied Energy*, vol. 239, pp. 70–82, 2019.

[9] B. Odetayo, J. MacCormack, W. Rosehart, and H. Zareipour, "A chance constrained programming approach to integrated planning of distributed power generation and natural gas network," *Electric Power Systems Research*, vol. 151, pp. 197–207, 2017.

[10] A. Ratha, J. Kazempour, A. Virag, and P. Pinson, "Exploring market properties of policy-based reserve procurement for power systems," in *2019 IEEE 58th Conference on Decision and Control (CDC)*, 2019, pp. 7498–7505.

[11] W. Wiesemann, D. Kuhn, and M. Sim, "Distributionally robust convex optimization," *Operations Research*, vol. 62, no. 6, pp. 1358–1376, 2014.

[12] R. D. Christie, B. F. Wollenberg, and I. Wangensteen, "Transmission management in the deregulated environment," *Proceedings of the IEEE*, vol. 88, no. 2, pp. 170–195, 2000.

[13] A. Georghiou, D. Kuhn, and W. Wiesemann, "The decision rule approach to optimization under uncertainty: Methodology and applications," *Computational Management Science*, vol. 16, no. 4, pp. 545–576, 2019.

[14] A. Georghiou, W. Wiesemann, and D. Kuhn, "Generalized decision rule approximations for stochastic programming via liftings," *Mathematical Programming*, vol. 152, no. 1, pp. 301–338, 2015.

[15] L. Roald and G. Andersson, "Chance-constrained AC optimal power flow: Reformulations and efficient algorithms," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 2906–2918, 2018.

[16] S. Zymler, D. Kuhn, and B. Rustem, "Distributionally robust joint chance constraints with second-order moment information," *Mathematical Programming*, vol. 137, no. 1, pp. 167–198, 2013.

[17] M. R. Wagner, "Stochastic 0-1 linear programming under limited distributional information," *Operations Research Letters*, vol. 36, no. 2, pp. 150 – 156, 2008.

[18] W. Xie and S. Ahmed, "Distributionally robust chance constrained optimal power flow with renewables: A conic reformulation," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1860–1867, 2018.

[19] Y. Dvorkin, "A chance-constrained stochastic electricity market," *IEEE Transactions on Power Systems (Early Access)*, 2019.

[20] C. Borraz-Sánchez, R. Bent, S. Backhaus, H. Hijazi, and P. Van Hentenryck, "Convex relaxations for gas expansion planning," *INFORMS Journal on Computing*, vol. 28, no. 4, pp. 645–656, 2016.

[21] G. P. McCormick, "Computability of global solutions to factorable nonconvex programs: Part I - Convex underestimating problems," *Mathematical Programming*, vol. 10, no. 1, pp. 147–175, 1976.

[22] A. Ratha, A. Schwele, J. Kazempour, P. Pinson, S. S. Torbaghan, and A. Virag, "Online appendix for paper: Affine policies for flexibility provision by natural gas networks to power systems," 2020. [Online]. Available: https://doi.org/10.5281/zenodo.3755120

[23] P. Pinson, "Wind energy: Forecasting challenges for its operational management," *Statistical Science*, vol. 28, no. 4, pp. 564–585, 11 2013.

[24] M. K. Singh and V. Kekatos, "Natural gas flow equations: Uniqueness and an MI-SOCP solver," in *2019 American Control Conference (ACC)*, 2019, pp. 2114–2120.

[25] C. Coffrin, H. L. Hijazi, and P. Van Hentenryck, "Strengthening the SDP relaxation of AC power flows with convex envelopes, bound tightening, and valid inequalities," *IEEE Transactions on Power Systems*, vol. 32, no. 5, pp. 3549–3558, 2017.

[26] J. E. Mitchell, J.-S. Pang, and B. Yu, "Convex quadratic relaxations of nonconvex quadratically constrained quadratic programs," *Optimization Methods and Software*, vol. 29, no. 1, pp. 120–136, 2014.