Hourly Pricing and Day-Ahead Dispatch Setting in Brazil: The DESSEM Model

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***Abstract*— Power generation planning and price setting in hydrothermal systems is a complex task, usually performed by a chain of optimization models, ranging from long to short-term time horizons. In Brazil, the weekly hydrothermal dispatch and spot pricing has been officially set since 2002 with two optimization models: NEWAVE and DECOMP. This work briefly presents the main features of a third model in this chain (DESSEM) which has been validated since 2017 by the Brazilian ISO and the Market Operator to determine the hourly dispatch and energy prices starting in 2020/2021. This model considers a very comprehensive set of thermal unit commitment constraints, nonconvex security constraints for the electrical network and a very detailed operation of the hydro plants. We apply CPLEX solver to the resulting MILP problem, with smart ad-hoc iterative procedures to reduce the computational burden. Results are presented for the large scale Brazilian system.**

***Index Terms*-- Hydrothermal scheduling, energy pricing, mixed-integer linear programming, unit commitment.**

I. Introduction

Power generation planning and price setting in large-scale hydrothermal systems is a complex task, usually performed by a chain of optimization models, ranging from long to short-term time horizons. Several examples of such type of coordination can be found in the literature, especially for predominantly hydro systems, where water values need to be carefully evaluated along time to allow an optimization of thermal and hydro resources [1]-[3]. In Brazil, the weekly hydrothermal dispatch and spot pricing has been officially set since 2000 with two optimization models: NEWAVE for mid/long term planning [4] and DECOMP [5], for the weekly dispatch. Both models represent many aspects of the entire integrated generation system, such as: interconnected system areas, power demand in load bocks, stochastic water inflows, anticipated dispatch of some thermal plants, and variation of productivity of the hydro plants with the water head.

Since 2017 the Brazilian Independent System Operator (ONS) and the market chamber (CCEE) have been validating a third model of this chain, labeled DESSEM, to determine the hourly dispatch and energy prices with a time horizon of up to one week and a time discretization of up to half-an-hour. The Brazilian government has approved the official use of such model by ONS starting in January 2020, to determine the day-ahead scheduling of the system, as well as its use from January 2021 by CCEE, to establish hourly prices in Brazil. Fig. 1 contains the main features of these optimization models and the rolling horizon scheme that is applied to coordinate them, by running them for each month *m* (NEWAVE), week *w* (DECOMP) and day *d* (DESSEM), with the time horizon and discretization shown in the table and updated information on reservoirs storages, inflows, load forecasts, etc.

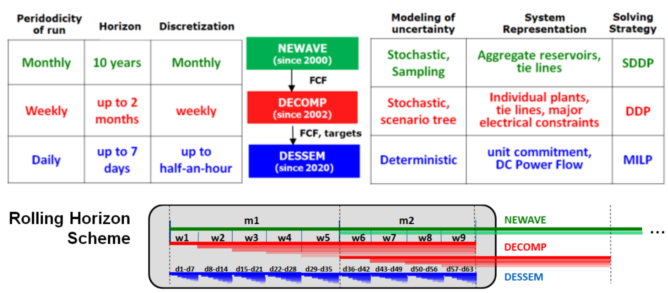


Figure 1. Main features and rolling horizon scheme for the optimization models used for hydrothermal operation planning in Brazil

This work briefly presents the main features of the DESSEM model and describes the results of the validation process that was conducted with the goal of launching the model for official use in 2020.

II. Network Constrained Hydrothermal Unit Commitment Model (NCHTUC)– Main Features

The main objective of the network constrained hydrothermal unit commitment model (NCHTUC) DESSEM is to determine the day-ahead operation of hydrothermal systems, also including new renewables such as wind and solar power. The optimization horizon is up to 1 week, with a time discretization of up to half-an-hour time steps. A so-called future cost function (FCF), which evaluates the value of water in the reservoirs and is provided by the mid-term model DECOMP is considered at the end of the time horizon. The model also yields nodal hourly prices, which are aggregated into system area prices for the energy market.

An individual representation of each reservoir and each hydro generation unit is employed, and the electrical network is represented by a DC model [6], taking into account line flow limits and additional security constraints, as detailed later. Thermal unit constraints are considered by a mixed-integer linear programming approach [7], including the operation constraints of combined cycle units [8]. Fig. 2 illustrates the basic system components and the time representation considered in the DESSEM model.

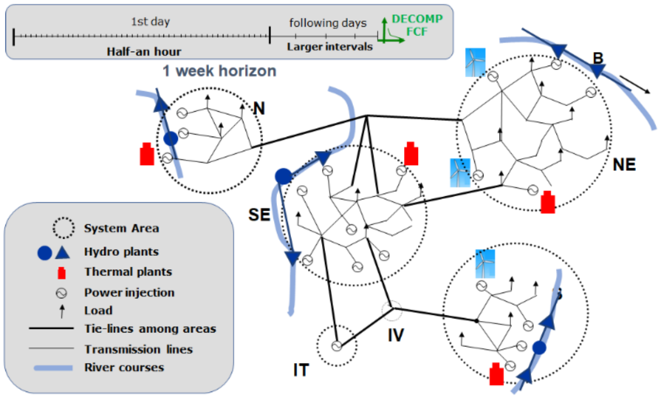


Figure 2. Time representation and system componentes in the NCHTUC model DESSEM.

Variation of efficiency of the hydro plants with the water head is represented through the four-dimensional piecewise linear model presented in [10], and an accurate representation of the hydro balance equations is modeled, taking into account a linear representation of river routing constraints [11] and also considering pumping stations, water channels among reservoirs and water level constraints at some river sections.

III. Mathematical Formulation

The NCHTUC problem is formulated in a cost minimization context as a mixed-integer linear programming problem (MILP), with static/dynamic models to represent nonlinear constraints [12], [13]. The next sections describe the main constraints of the problem, where the upper index $t = 1, \dots, T$ indicates the time steps along the planning horizon.

A. Objective function

The objective function (1) is to minimize total system operation costs Z, given by:

$$Z = \sum_{t=1}^{T}\left[\sum_{i=1}^{nt}(cst_i^t + ct_i^t gt_i^t) + \sum_{i=1}^{NCI} ci_i^t Eim_i^t - \sum_{i=1}^{NCE} ce_i^t Eex_i^t\right] + \alpha_{FCF}(V_i + R_i) \qquad (1)$$

where nt is the number of thermal units, each one with a generation gt_i^t subject to linear fuel costs ct_i^t , and NCI (NCE) are the number if import (export) contracts with external systems, in quantity × price bids given by the pairs (Eim_i^t, ci_i^t) and (Eex_i^t, ce_i^t), respectively. The so-called "status-change" cost cst_i^t for thermal units comprises both startup and shutdown costs, according to expression (2):

$$cst_i^t \ge C_i^{cold} \cdot (u_i^t - u_i^{t-1}) \qquad (2)$$
$$cst_i^t \ge C_i^{shut} \cdot (u_i^{t-1} - u_i^t)$$

where C_i^{cold} (C_i^{shut}) are fixed startup (shutdown) costs incurred in time step t whenever the status u of the unit changes from 0 to 1 (startup) or from 1 to 0 (shutdown), between time steps (t -1) and t. We note that only one of these two procedures can be active at a time, which allows us to use a single variable to denote both costs.

Finally, α_{FCF} is the expected future operation cost, which depends on the hydrological conditions given by the vector of storages V_i^T in the reservoirs and the amount of water in the river courses R_i^T (see [11] for details) at the end of the scheduling horizon. Such cost is given by a piecewise linear function (3) with $NCUT_{FCF}$ provided by the mid-term model, where π_{FCF,V_i}^k is the term related to the storage in reservoir i in cut k and $\pi_{FCF_0}^k$ is the independent term:

$$\alpha_{FCF} \ge \pi_{FCF_0}^k + \sum_{i=1}^{NH} \pi_{FCF,V_i}^k . (V_i^T + R_i^T), \qquad k = 1, NCUT_{FCF} \qquad (3)$$

B. Constraints already considered in the mid-term model

The set of constraints described in this section are already present in the mid-term model DECOMP and only adjusted or improved in DESSEM according to its more refined time discretization. We refer to [5] for more details about them.

1) Power balance equations

Constraint (4) is the power balance equation to meet the demand D_k^t in each system area k, which is composed by the sum of the loads d_i^t in all buses belonging to area k. Variables GH_i^t, gt_i^t and $G_{w_i}{}^t$ are the outputs of each hydro plant, hydro unit or wind farm i, and Qp_i^t is the flow in each pumping station with consumption rate $Cons_{PS_i}$ (MW/(m³/s)). Interconnections Int_{ik}^t with other neighbor system areas in the set Ω_{SI_k} are also considered. Generation in other small sources, whose operation is determined in a decentralized way, are known values and subtracted from the load. Finally, the sets Ω_{SH_k}, Ω_{ST_k} Ω_{SW_k}, Ω_{SP_k}, Ω_{SCI_k}, Ω_{SCE_k} and Ω_{SO_k} comprise the components of each type that belongs to area k.

$$\sum_{i \in \Omega_{SH_k}} GH_i^t + \sum_{i \in \Omega_{ST_k}} gt_i^t - \sum_{i \in \Omega_{SP_k}} Cons_{PS_i} Qp_i^t + \sum_{i \in \Omega_{SW_k}} G_{w_i}{}^t - \sum_{i \in \Omega_{SCE_k}} Eex_i^t + \sum_{i \in \Omega_{SCI_k}} Eim_i^t + \sum_{i \in \Omega_{SI_k}} \left(Int_{ik}^t - Int_{kj}^t\right) = D_k^t - \sum_{i \in \Omega_{SO_k}} G_{SO_i}{}^t \qquad (4)$$

We note that each variable x_i^t has an upper bound $\overline{x_i^t}$ given by maximum physical/operation limits, except for the value $\overline{G_{w_i}{}^t}$, which corresponds to the forecast value of wind generation of each wind farm, which can be curtailed if necessary or economical, as presented in [14].

2) Hydro balance equations

Hydro balance in the reservoirs is formulated as (4):

$$V_i^t - V_i^{t-1} + \varsigma^t \left[+\left(Q_i^t + S_i^t + {Q_{ev}}_i^t\right) + \sum_{j \in {M_p}_i} {Q_b}_j^t \right.$$

$$\left. - \sum_{j \in {J_p}_i} {Q_p}_j^t - \sum_{j \in M_i} \left(Q_j^t + S_j^t\right) - \sum_{j \in {M_{tv}}_i} \left(Q_j^{t-\tau_{ji}} + S_j^{t-\tau_{ji}}\right) \right]$$

$$= \varsigma^t \left[I_i^t - {Q_{out}}_i^t \right] \tag{5}$$

where the known values are the natural inflows I_i^t to reservoirs and water intakes ${Q_{out}}_i^t$ for other uses of water. Variables Q_i^t and S_i^t are the turbined (discharge) and spilled outflows and ${Q_{ev}}_i^t$ is the evaporation in the reservoir, modeled as a linear function (5) of the average storage in t, with ${k_{evap}}_i^0$ and ${k_{evap}}_i^1$ coefficients computed by linear regression:

$$ {Q_{ev}}_i^t = {k_{evap}}_i^0 + {k_{evap}}_i^1 (V_i^t + V_i^{t-1})/2 \Big] \tag{6}$$

The sets ${M_p}_i/{J_p}_i$ indicate upstream/downstream pumping stations that take water from/to reservoir i, and ${M_{tv}}_i/M_i$ are the set of upstream plants j with/without water delay time to reservoir i, with a value of τ_{ji} in the first case. The factor ς^t converts m^3/s to hm^3. Additional aspects related to hydro balance constraints that are considered in the model, as for example flows in water channels, bypasses between reservoirs and propagation of water along the river basins [11] are not detailed here due to lack of space.

3) Hydro production function

The hydro generation for plant i is a concave piecewise-linear function composed of NPF_i cuts indexed by k, with coefficients ${\gamma_V}_i^k$, ${\gamma_Q}_i^k$ and ${\gamma_S}_i^k$ related to storage in the reservoir, turbined and spilled outflows, respectively, plus an independent term ${\gamma_0}_i^k$, as shown in (7). A detailed description on how to build this function can be found in [10].

$$\begin{cases} GH_i^t \le {\gamma_0}_i^k + {\gamma_V}_i^k V_i^t + {\gamma_Q}_i^k Q_i^t + {\gamma_S}_i^k S_i^t \\ \quad t = 1, \dots, T, k = 1, \dots, NPF_i \end{cases} \tag{7}$$

4) Additional operating constraints

There are several operation constraints for the hydro plants similar to the ones described in [5], adapted to the daily scheduling context. Moreover, ramp constraints for all hydro-related variables (GH, V, Q, S) are considered in the model, including also the water level in some sections along the river courses, as described in [11].

C. Thermal unit commitment constraints

A detailed operation of thermal units is considered, mainly based in the formulations proposed in [7], [8], [15].

1) Minimum generation and startup/shutdown curves

These constraints impose minimum and maximum generation limits once the unit is on. However, during the startup/shutdown trajectories the power of the unit may be below such minimum value and should follow pre-defined curves. This aspect is represented through constraints (8)-(9), based on the work [15], where NUp_i (NDn_i) is the number of steps of the startup (shutdown) curve, and $\{TrUp_i^k, k = 1, \dots, NUp_i\}$ ($\{TrDn_i^k, k = 1, \dots, NDn_i\}$) are the power values for each segment of such curves. We apply continuous auxiliary variables $\hat{y}_i^t$, $\breve{y}_i^t \in [0,1]$ that indicate the unit is under a startup or shutdown trajectory in time step t. The effect of startup/shutdown trajectories is shown in Fig. 3.

$$gt_i^t \ge \underline{gt}_i \left(u_i^t - \sum_{k=1}^{NUp_i} \hat{y}_i^{t-k+1} - \sum_{k=1}^{NDn_i} \breve{y}_i^{t+k-1} \right) + $$

$$+ \sum_{k=1}^{NUp_i} TrUp_i^k \, \hat{y}_i^{t-k+1} + \sum_{k=1}^{NDn_i} TrDn_i^{NDn_i-k+1} \breve{y}_i^{t+k-1} \tag{8}$$

$$gt_i^t \le \overline{gt}_i \left(u_i^t - \sum_{k=1}^{NUp_i} \hat{y}_i^{t-k+1} - \sum_{k=1}^{NDn_i} \breve{y}_i^{t+k-1} \right) + $$

$$+ \sum_{k=1}^{NUp_i} TrUp_i^k \cdot \hat{y}_i^{t-k+1} + \sum_{k=1}^{NDn_i} TrDn_i^{NDn_i-k+1} \cdot \breve{y}_i^{t+k-1} \tag{9}$$

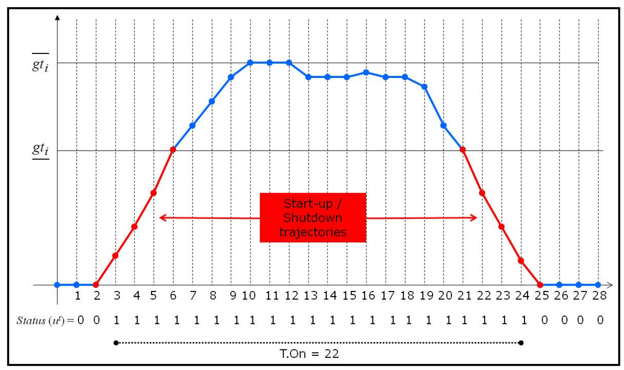


Figure 3. Example of startup/shutdown trajectories of thermal units.

2) Maximum up and down ramp rates

Constraints (10)-(11) impose maximum ramp-rates RUp_i (RDn_i) for increase/decrease of generation of thermal units in consecutive time steps. Indicator variables $\hat{y}$ and $\breve{y}$ are useful to deactivate them during startup/shutdown processes.

$$gt_i^t - gt_i^{t-1} \le RUp_i +$$

$$\left(\overline{gt}_i - RUp_i\right) \left(\sum_{k=1}^{NUp_i} \hat{y}_i^{t-k+1} + \sum_{k=1}^{NDn_i} \breve{y}_i^{t+k-1} \right) \tag{10}$$

$$-gt_i^{t+1} + gt_i^t \le +$$

$$RDn_i + \left(\overline{gt}_i - RDn_i\right) \cdot \left(\sum_{k=1}^{NUp_i} \hat{y}_i^{t-k+1} + \sum_{k=1}^{NDn_i} \breve{y}_i^{t+k-1} \right) \tag{11}$$

3) Minimum up/down times

Constraints (12)-(13) force the units to be on (off) for $\mathrm{Ton_i}$ ($\mathrm{Toff_i}$) time steps after a startup/shutdown process:

$$\sum_{k=t}^{t+Ton_i} u_i^k \geq Ton_i(u_i^t - u_i^{t-1}) \tag{12}$$

$$\sum_{k=t}^{t+Toff_i} \left(1 - u_i^k\right) \geq Toff_i(u_i^{t-1} - u_i^t) \tag{13}$$

4) Modeling of combined cycle units

Combined cycle plants are composed of a mix of steam and gas turbines that operate in a coordinate way to increase the overall plant efficiency, as shown in Fig. 4, taken from [8].

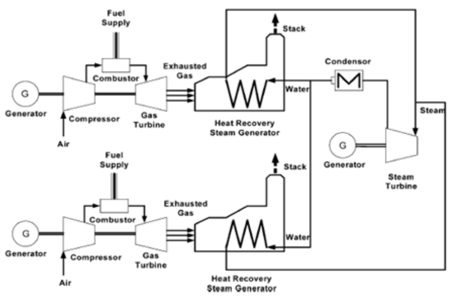


Figure 4. Illustrative scheme of a combined-cycle plant (source: [8]).

The complexity in modeling such constraints led to several approaches in the literature, varying from simpler forms (an equivalent model for the whole plant) to more complex ones (an explicit representation of each unit). We consider the "configuration-mode" concept [8], where each configuration i is modeled as a different unit. All constraints presented in the previous subsections are employed, with the difference that startup/shutdown costs for each configuration are replaced by transition costs between different configurations. We also included, constraint (14) that forces only one configuration to be active at each time step, as well as constraints (15) that limit the variation of generation of the whole plant to a value $RTrans_j$ when there is a switch in configuration.

$$\sum_{i \in \Omega_{CC_j}} u_i^t \leq 1, \tag{14}$$

$$\left| \sum_{i \in \Omega_{CC_j}} gt_i^t - \sum_{i \in \Omega_{CC_j}} gt_i^{t-1} \right| \leq RTrans_j, \tag{15}$$

for each combined-cycle plant j with units in the set Ω_{CC_j}.

D. Detailed electrical network constraints

We consider a dc power flow model for the electrical network, which allows the use of linear programming to represent active power flow limit constraints $\overline{f_l}$ in each transmission line l. We employ participation factors ${\kappa_B}_i^l$ for the injection of bus i in line l to represent such constraints as in (16), where g_i^t denotes the total generation in bus i, coming from any of the components expressed in power equation (4).

$$-\overline{f_l} \leq \sum_{i=1}^{NB} {\kappa_B}_i^l \, [g_i^t - d_i^t] \leq \overline{f_l} \tag{16}$$

In addition, security constraints i on the mixed sum of power injections on buses in the set SCB_i and power flows of transmission lines in the set SCL_i are also enforced in (17). We note that to keep the size of the problem moderate, both types of constraints (16) and (17) are included "on demand", i.e., only when needed to ensure a feasible solution, as described in reference [6].

$$\underline{SC_i^t} \leq \sum_{b \in SCB_i} {\kappa_{SCB}}_i^b (g_b^t - d_b^t) + \sum_{l \in SCL_i} {\kappa_{SCL}}_i^l f_l^t \leq \overline{SC_i^t}. \tag{17}$$

In order to represent dynamic limits for the flows in some tie-lines of the system, with the aim to to ensure stability of the electrical network in the real system operation, additional electric constraints are included as follows:

1) Constraints limits given by tables

The flow limit in some tie-lines may vary with the load of a system area and the flows in other transmission lines, which are unknown *a priori*, since they depend on the generation dispatch that is yet to be obtained. Fig, 5 below illustrates a constraint, set by the Brazilian ISO, for the maximum flow in the tie-line labeled as "RNE", depending on the load in area NE, the export flow from area *N*, and the sum *F* in other two lines of the system.

Bounds on RNE tie-line (MW)						
	NE load < 10,500		10,500 < NE load < 12,000		NE load > 12,000	
Export from system area N	F: sum of flows in tie-line Igapora III and tie-line Igaipora II/Bom Jesus da Lapa II					
	0 < F ≤ 600	600 < F ≤ 1.050	0 < F ≤ 600	600 < F ≤ 1.050	0 < F ≤ 600	600 < F ≤ 1.050
Exp_N ≥ 5.000		Limite = 40% da carga NE	4.400 (1)	4.300 (1)	4.400	4.300
4.000 ≤Exp_N< 5.000			4.300 (1)	4.300 (1)	4.300	4.300
3.000 ≤Exp_N< 4.000		4.200 (1)	4.200 (1)	4.200 (1)	4.200	4.200
2.000 ≤Exp_N< 3.000		4.100 (1)	4.100	4.100	4.100	4.100
1.000 ≤Exp_N< 2.000		3.900 (1)	4.000	3.900	4.000	3.900
0 ≤Exp_N< 1.000	4.100 (1)	3.600 (1)	4.000	3.700	4.000	3.500
0 < RN ≤ 500	3.900 (1)	3.300 (1)	4.000	3.500	4.000	3.500
500 < RN ≤ 1.000	3.600 (1)	3.000 (1)	3.800	3.200	4.000	3.300
1.000 < RN ≤ 1.500	3.200 (1)	2.700 (1)	3.400	2.900	3.600	2.900

Figure 5. Example of dynamic limits of a security constraint that are defined by a table.

Such limits are included in the model in an iterative way, as shown in the flow chart of Fig. 6.

2) Constraints limits given by piecewise linear functions

If the shape of the function that defines the dynamic limits of some security constraints based on the values of other line flows is nearly "stair-wise concave" (i.e., the midpoints of each step tend to show a concave shape), it is possible to employ a lower piecewise linear concave approximation for this function. Therefore, we avoid the iterative process shown in Fig. 6, which increases the computational burden to solve the problem. An example of such curve (in blue) and its lower piecewise linear approximation (in red) is shown in Fig. 7.

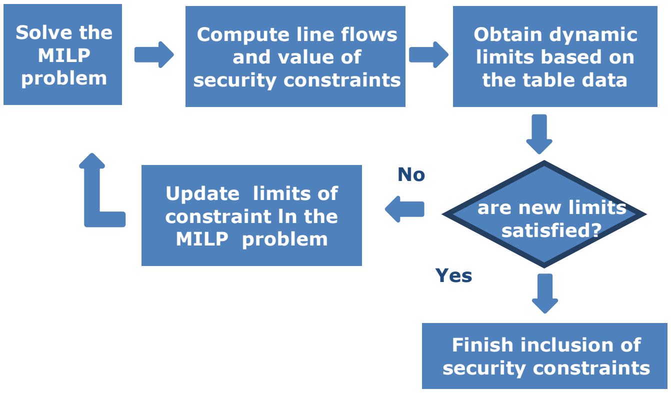


Figure 6. Iterative process to consider dynamic limits of security constraints (16) that are given by tables.

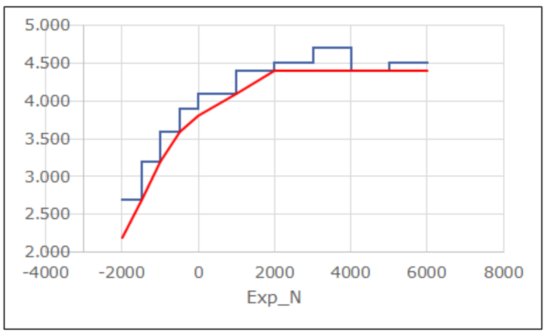


Figure 7. Piecewise linear approximation of network security constraints.

The mathematical formulation of these piecewise-linear constraints is presented in (18), (19), where SC_i^t is the value of the constraint (the middle term in (16)) and $\overline{SC_{LPP}}_i^t$ is the limit of the constraint, obtained as the lowest value of all linear approximations k=1,...,$NCUT_{SCLPP_i}$, whose coefficients $\kappa_{SCLPP_0}{}_i^k$ and $\kappa_{SCLPP_P}{}_i^k$ are applied to the value of the other security constraint $contr(i)$ that controls security constraint i.

$$SC_i^t \leq \overline{SC_{LPP}}_i^t \tag{18}$$

$$\overline{SC_{LPP}}_i^t \leq \kappa_{SCLPP_0}{}_i^k + \kappa_{SCLPP_P}{}_i^k SC_{contr(i)}^t, \tag{19}$$

E. Remarks

The following remarks can be made regarding the problem formulation and the generality of the proposed approach in other contexts:

- a deterministic problem has been considered, since the share of intermittent generation in the Brazilian system is very small (around 7%). However, the formulation could be extended by considering several scenarios of combined wind power and hydro inflows, where the first stage solution could be the commitment status of thermal units and economic dispatch constraints could be included for each scenario;

- most features mentioned throughout this section could also be extended to a market-oriented or profit maximization framework, with proper adjustments. For example, in the first case, instead of considering cost curves for the thermal plants and the future cost function (3) to assess the value of water, one could use piecewise linear price bids for all types of generators. Moreover, the self-scheduling of a plant in a profit-oriented context could be considered, once some boundary constraints and conditions are defined for the operation of the other resources and the transmission system;

IV. Solution Approach

In this section the solution approach to solve the NCHTUC problem, modeled as a multi-stage mixed-integer linear program, is detailed. In terms of mathematical dimension, a typical instance of our model, which is run daily by the Brazilian Independent System Operator, has approximately one hundred thousand binary variables, half a million of continuous ones and half a million of constraints. The iterative process presented in Fig.6 is an exact approach to solve this problem if the MILP problems are solved to optimality through a branch-and-cut algorithm. Even when network constraints (15) and (16) are included only when necessary, since most of them are not binding, such kind of instance becomes unpractical to be solved in some cases, since it can takes several hours to solve the problem.

In order to be used by the ONS to determine the hourly hydrothermal dispatch in each day, the solution time must be limited in a maximum of two hours. Due to this reason we focus our efforts in a procedure to find a high quality, possible near optimal, mixed-integer solution in this amount of time, and have developed a novel approach following the ideas presented in [18]. The aim is to determine lower and upper limits for the problem, but instead of applying the Local Branching (LB) [20] technique, a modified version of Feasibility Pump (FP) [19] together with the branch-and-cut algorithm available in IBM/CPLEX was employed.

The LB technique, which was employed in [18], is very efficient when an initial integer feasible solution (warm start) is available, so the sequence of MIPs refines it to a high quality feasible integer solution. However, in this real large-scale instance we have observed that even finding a warm start solution is a hard computational task. Due to this reason we have applied a modified FP approach that employs a sequence of linear programs solved through Interior Point Method and adding in the objective function a term resembling the Hamming's distance, which measures the total number of replacements needed to make equal two binary status vectors.

The use of Interior Point Method (IPM) is essential in this approach since all sequence of linear programs, used in the process of adding networks constraints and/or in FP can be solved in few minutes. We note that one of reasons we have exploit an extended and sparse unit commitment formulation instead of compact and novel ones like [9], [21]-[23], is that IPM behaves efficiently exactly in such kind of structure [24].

The overall iterative process is described in Fig. 8 and yields a high quality commitment for the thermal units and generation dispatch for the system. A lower bound (L_{inf}) for the NCHTUC problem is obtained with the linear relaxation of the problem solved through IPM in a framework to consider violated network constraints. The upper bound (L_{sup}) is also computed after fixing the commitment status coming from the *Algorithm ALG -FP*. The quality of the feasible integer solution is measured as the relative gap between L_{inf} and L_{sup} and, in almost all cases, is close to optimality.

A. Algorithm ALG-FP

This algorithm finds a feasible integer solution for the NCHTUC problem with the network constraints added so far.

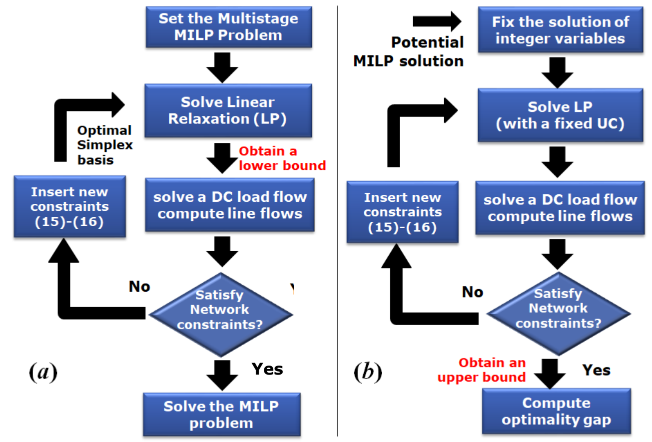
- **Step 0**: relax the whole set of commitment status and solve the problem with IPM method, obtaining a feasible continuous commitment status vector $\hat{u}$. Set FP_{max} the iteration limit;
- **Step 1**: if vector $\hat{u}$ is integer then STOP. Otherwise rounds $\hat{u}$ to the closest integer vector $\bar{u}$;
- **Step 2**: considers in the objective function, for each thermal unit, the term $-\sum_{\{v:(\bar{u})^v=1\}} P\cdot(u)^v+\sum_{\{v:(\bar{u})^v=0\}} P\cdot(u)^v$, where $(u)^v$ is the coordinate v of a vector u and P is a penalty parameter to impose significance to commitment status;
- **Step 3**: solve the problem through IPM method, obtaining the feasible continuous commitment status vector $\hat{u}$. If FP_{max} has not been attained go to **Step 1**;
- **Step 4**: (fixing status step) for all thermal unit, if all coordinates in vector $\hat{u}$ are 1 (0) then turn on (turn off) the respective unit in the problem;
- **Step 5**: return to the original objective function and solve the resulting MIP by a branch-and-cut algorithm obtaining the feasible integer commitment status vector $\hat{u}$. STOP.

One should note that the maximum number of FP iterations may limit the possibility to reach an integer solution at the end of the sequence of linear programs. However, we can heuristically use this quasi-integer solution found so far to fix the commitment status of several thermal units in the final MIP, to reduce computational complexity.

B. Energy prices for the market

One major issue of the NCHTUC problem is how to obtain system marginal costs that can be used to price energy for the market, since dual information is not readily available when solving a MILP problem. We apply the approach proposed in [17] where unit status are fixed after obtaining a solution for the MILP problem and a final linear program (LP) is solved to obtain the marginal prices. Moreover, since the dc power flow is not explicitly embedded in the problem, nodal prices in each bus are not directly obtained. Rather, we compute it after solving the LP with the multipliers λ of all constraints with bus loads terms on the right hand side, for each time step t:

- ${\lambda_k^D}^t$: power demand equation (3) in area k related to bus i;
- ${\lambda_i^l}^t$: flow limit constraints (15) of each line i
- ${\lambda_i^{SC}}^t$: i^{th} security constraint (16)
- ${\lambda_{i,k}^{SCLPP}}^t$: k^{th} cut for the i^{th} piecewise linear model (18) for dynamic limits of security constraints.



Iterative process to include violated network constraints (a) and to later find a high quality, near optimal, solution to MILP problem (b).By applying the corresponding participation factors of bus loads in each of the above constraints, we obtain the final expression for the marginal cost BMC_i^t in each bus i:

$$BMC_i^t = {\lambda_k^D}^t + \sum_{l=1}^{NL} {\lambda_i^l}^t\left(-{\kappa_B}_i^l\right) + \\ + \sum_{j=1}^{NSC} {\lambda_i^{SC}}^t \left({\kappa_{SCB}}_j^i + \sum_{l'\in SCL_j}\left(-{\kappa_B}_i^{l'}\right){\kappa_{SCL}}_j^{l'}\right) + \\ + \sum_{j=1}^{NLPP} \sum_{k=1}^{NCUT_{SCLPP_i}} {\lambda_{i,k}^{SCLPP}}^t \left({\kappa_{SCB}}_{contr(j)}^i + \sum_{l'\in SCL_{contr(j)}} \left(-{\kappa_B}_i^{l'}\right){\kappa_{SCL}}_{contr(j)}^{l'}\right) \tag{20}$$

Finally, if energy prices are set by market area (which is the case of Brazil) instead of nodal prices, we obtain the price AMC_i^t for each area k as an average of marginal costs of the buses located in this area (21), indicated by the set ${\Omega_{SB}}_k$, weighted according to their corresponding loads (20):

$$AMC_i^t = \sum_{i\in\Omega_{SB_k}} \left(CMB_i^t d_i^t\right) \Big/ \sum_{k\in\Omega_{SB_k}} d_i^t \tag{21}$$

V. NUMERICAL RESULTS

The optimization model presented in this paper has been officially tested since January 1st, 2019 by both the ISO and the market operator, in order to assess the behavior of the hourly energy prices prior to the official use of the model in 2020. We present the main results obtained with this model by a set of 55 test cases, each one for the day-ahead scheduling of the real large scale Brazilian system for a given day from January to April 2019.

A. System Characteristics

The main dimensions of the Brazilian system and number of constraints (per time step) are given in Table 1 below:

TABLE I. RELEVANT PARAMETERS OF THE BRAZILIAN SYSTEM

System areas	4	Combined-cycle plants	9
Hydro plants	162	buses	6746
Thermal units	438	Transmission lines	9213

B. *Illustrative operation Results*

Since it is not possible to show detailed operation results due to lack of space, we show examples of two features that are considered in the model.

1) *Dynamic limits for security constraints*

We use as an example the imported energy by the NE area of Brazil, whose limits depend on the flow in one major tie-line, the load and the amount of generation of intermittent sources (wind and solar) in this area. There are limits both given by table (Fig. 5) and by a piecewise linear approximation (Fig. 7) for this flow, whose values for the first day are shown in Fig. 9. We note that the constraint becomes binding since approximately 8am until the end of the day.

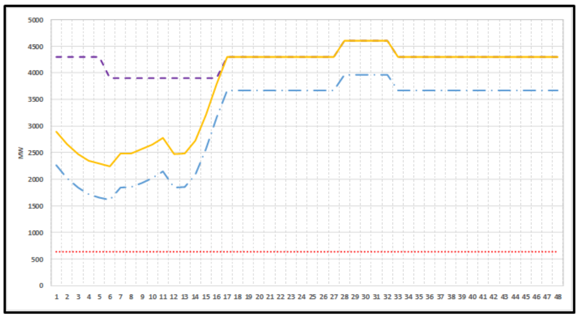


Figure 8. Example of the value of a given security constraint (yellow line), whose limit (upper dashed line) is dnamic

2) *Operation of thermal units*

In Fig. 10 we show the operation of a given thermal unit, which is off at the beginning of the scheduling horizon and whose incremental cost is lower that the area marginal price obtained in the final solution. Since the startup cost of this unit pays off for the economy in the system generation cost when using this unit, it is turned on at the beginning of the day and has to follow a strict startup trajectory as formulated in (9), until its generation remains within its corresponding lower/upper bounds, shown in red.

C. *Performance of the model*

In this section we assess the performance of the iterative approach proposed in Fig. 8 to solve the MILP problem in crucial aspects that should be assessed in its official use for the Brazilian system. We run 241 official test cases set by the ISO from January to August 2019, using AMD processors with 96 GBytes of RAM memory. Version 12.9 of the CPLEX solver was employed, with parallel processing.

1) *CPU time*

The average CPU time for each test case was 41 minutes, which is quite reasonable due to the size of the problem. The cumulative distribution of CPU times is shown in Fig.11, where we note that 70% of the cases took less than 1 hour to be solved and only 5% of the cases took more than 2 hours.

2) *Assessment of Optimality*

We solved 55 test cases comparing two solving strategies: either by solving through the exact approach or by applying our novel approximate iterative procedure. The lack of optimality of our approach as compared to the direct method is shown in Fig. 12. The maximum deviation was 0.008%, and in some cases we even found a better solution, which is due to the optimality tolerance (0.1%) set to the MILP solver.

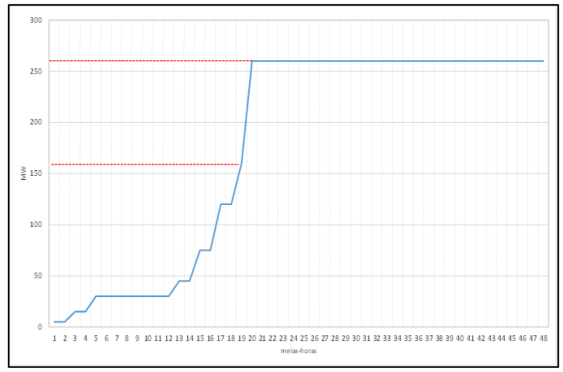


Figure 9. Example of a startup process of a thermal unit.

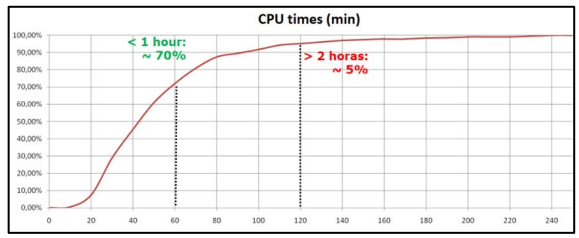


Figure 10. CPU time to solve the NCHTUC.

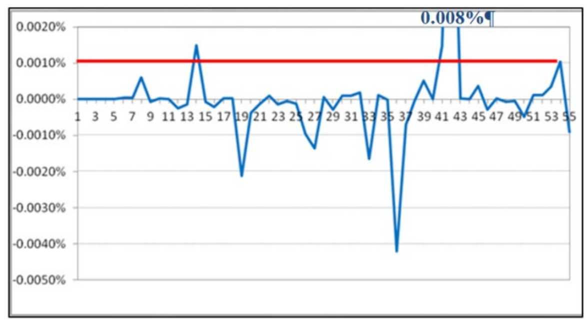


Figure 11. Assessment of optimality of the proposed approach, as compared to directly solving the problem as a big MILP.

Moreover, in order to provide an overview of our approach in terms of CPU time we detail in Table 2 six real instances built by ONS in 2019, which were randomly chosen among our portfolio of cases. The branch-and-cut algorithm used in both approaches was set 0.1% of optimality gap. In all instance our iterative converge to a near optimal integer feasible solution. In Table 2, #Cols is the number of columns, #Rows is the number of constraints, #Bin is the number of binaries.

TABLE II. COMPUTATION PERFORMANCE

Instance	#Cols	#Rows	#Bin	Proposed approach	Exact approach
March 12th	412,621	359,663	104,460	0h:43min	3h:00min
April 1st	440,705	385,618	111,488	1h 12min	0h:50min
April 7th	462,197	408,039	118,456	1h:02min	1h 16min
August 24th	484,008	423,100	122,262	1h 14min	6h 30min
November25th	504,705	366,900	110,208	0h:33min	1h 05min
November26th	473,221	342,823	103,320	0h:32min	1h 12min

Based on the results we can see that our proposed iterative approach outperforms the exact one in most of cases and made possible the official use of DESSEM model since it could accomplish the limit of two hours established by ONS, as we can see for example in the case of 2019 August 24th.

3) Marginal costs

Fig. 13 shows the marginal costs for the *NE* area with the proposed approach (blue) for 28 days in February, as compared to the direct resolution of the problem. The values are almost identical, with larger differences in some hours.

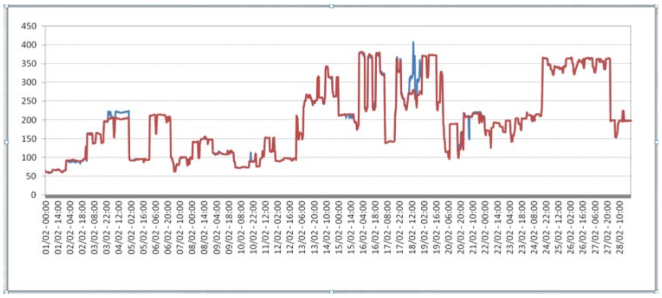


Figure 12. Marginal costs for the NE area during 28 runs of the model for Feb 2019, for the direct (red) and the proposed iterative strategy (blue).

VI. CONCLUSIONS

This paper presents a short term network constrained hydrothermal unit commitment model for large scale systems with very detailed hydro, thermal and network constraints. This model was validated by the independent system operator and the market operator for official use in 2020 and 2021, respectively, to set the hourly dispatch and marginal prices in Brazil. Numerical results show the high performance of the proposed iterative approach to solve such a complex MILP problem to near optimality, in a reasonable CPU time. As future works, we mention the consideration of uncertainties in the generation of intermittent sources, such as wind/ solar plants, and the assessment of the performance of this approach with novel and/or more tight and compact formulations of the thermal unit commitment constraints, which have been presented for pure thermal systems [21]-[23] but whose performance are yet to be tested for systems with a large number of hydro reservoirs, a more detailed modeling of the electric network including security constraints, and additional spatial/temporal constraints for the components of the system.

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